# INSTRUCTOR'S SOLUTIONS MANUAL

MATTHEW G. HUDELSON

# BASIC TECHNICAL MATHEMATICS

AND

# BASIC TECHNICAL MATHEMATICS WITH CALCULUS ELEVENTH EDITION

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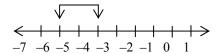
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#### Chapter 1

### **Basic Algebraic Operations**

#### 1.1 Numbers

- 1. The numbers –7 and 12 are integers. They are also rational numbers since they can be written as  $\frac{-7}{1}$  and  $\frac{12}{1}$ .
- 2. The absolute value of -6 is 6, and the absolute value of -7 is 7. We write these as |-6| = 6 and |-7| = 7.
- 3. -6 < -4; -6 is to the left of -4.



- 4. The reciprocal of  $\frac{3}{2}$  is  $\frac{1}{3/2} = 1 \times \frac{2}{3} = \frac{2}{3}$ .
- 5. 3 is an integer, rational  $\left(\frac{3}{1}\right)$ , and real.
  - $\sqrt{-4}$  is imaginary.
- **6.**  $\frac{\sqrt{7}}{3}$  is irrational (because  $\sqrt{7}$  is an irrational number) and real.
  - -6 is an integer, rational  $\left(\frac{-6}{1}\right)$ , and real.
- 7.  $-\frac{\pi}{6}$  is irrational (because  $\pi$  is an irrational number) and real.
  - $\frac{1}{8}$  is rational and real.
- 8.  $-\sqrt{-6}$  is imaginary.
  - $-2.33 = \frac{-233}{100}$  is rational and real.
- **9.** |3| = 3

$$\left|-3\right|=3$$

$$\left|-\frac{\pi}{2}\right| = \frac{\pi}{2}$$

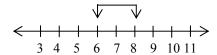
**10.** |-0.857| = 0.857

$$\left|\sqrt{2}\right| = \sqrt{2}$$

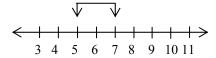
$$\left| -\frac{19}{4} \right| = \frac{19}{4}$$

#### 2 Chapter 1 Basic Algebraic Operations

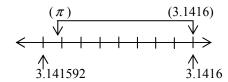
11. 6 < 8; 6 is to the left of 8.



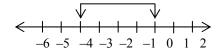
12. 7 > 5; 7 is to the right of 5.



**13.**  $\pi$  < 3.1416;  $\pi$  (3.1415926...) is to the left of 3.1416.



**14.** -4 < 0; -4 is to the left of 0.

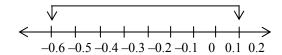


**15.** -4 < -|-3|; -4 is to the left of -|-3|, (-|-3|=-(3)=-3).

**16.**  $-\sqrt{2} > -1.42$ ;  $(-\sqrt{2} = -(1.414...) = -1.414...), -\sqrt{2}$  is to the right of -1.42.

17.  $-\frac{2}{3} > -\frac{3}{4}; -\frac{2}{3} = -0.666...$  is to the right of  $-\frac{3}{4} = -0.75$ .

**18.** -0.6 < 0.2; -0.6 is to the left of 0.2.



19. The reciprocal of 3 is  $\frac{1}{3}$ .

The reciprocal of  $-\frac{4}{\sqrt{3}}$  is  $-\frac{1}{4/\sqrt{3}} = -\frac{\sqrt{3}}{4}$ .

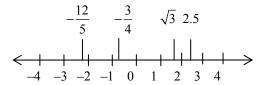
The reciprocal of  $\frac{y}{b}$  is  $\frac{1}{y/b} = \frac{b}{y}$ .

**20.** The reciprocal of  $-\frac{1}{3}$  is  $-\frac{1}{1/3} = -\frac{3}{1} = -3$ .

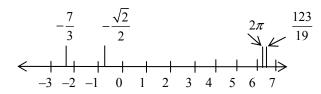
The reciprocal of  $0.25 = \frac{1}{4}$  is  $\frac{1}{1/4} = \frac{4}{1} = 4$ .

The reciprocal of 2x is  $\frac{1}{2x}$ .

**21.** Find 2.5,  $-\frac{12}{5} = -2.4; -\frac{3}{4} = -0.75; \sqrt{3} = 1.732...$ 



**22.** Find  $-\frac{7}{3} = -2.333...; -\frac{\sqrt{2}}{2} = -\frac{1.414...}{2} = -0.707; \ 2\pi = 2 \times 3.14... = 6.28; \ \frac{123}{19} = 6.47.$ 



- 23. An absolute value is not always positive, |0| = 0 which is not positive.
- **24.** Since  $-2.17 = -\frac{217}{100}$ , it is rational.
- 25. The reciprocal of the reciprocal of any positive or negative number is the number itself. The reciprocal of n is  $\frac{1}{n}$ ; the reciprocal of  $\frac{1}{n}$  is  $\frac{1}{1/n} = 1 \cdot \frac{n}{1} = n$ .
- **26.** Any repeating decimal is rational, so  $2.\overline{72}$  is rational. It turns out that  $2.\overline{72} = \frac{30}{11}$ .
- 27. It is true that any nonterminating, nonrepeating decimal is an irrational number.

#### 4 Chapter 1 Basic Algebraic Operations

**28.** No, |b-a| = |b| - |a|, as shown below.

If 
$$a > 0$$
, then  $|a| = a$ .

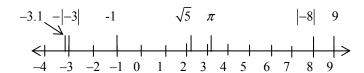
If 
$$b > a$$
 and  $a > 0$ , then  $|b| = b$ .

If 
$$b > a$$
 then  $b-a > 0$ , then  $|b-a| = b-a$ .

Therefore, 
$$|b-a| = b - a = |b| - |a|$$
.

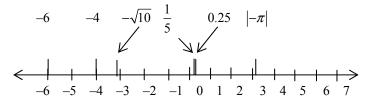
The two sides of the expression are equivalent, one side is not less than the other.

**29.** List these numbers from smallest to largest: -1, 9,  $\pi = 3.14$ ,  $\sqrt{5} = 2.236$ , |-8| = 8, -|-3| = -3, -3.1.



So, from smallest to largest, they are -3.1, -|-3|, -1,  $\sqrt{5}$ ,  $\pi$ , |-8|, 9

**30.** List these numbers from smallest to largest:  $\frac{1}{5} = 0.20$ ,  $-\sqrt{10} = -3.16...$ , -|-6| = -6, -4, 0.25,  $|-\pi| = 3.14...$ 



So, from smallest to largest, they are  $-\left|-6\right|, -4, -\sqrt{10}, \frac{1}{5}, 0.25, \left|-\pi\right|.$ 

- **31.** If a and b are positive integers and b > a, then
  - (a) b-a is a positive integer.
  - **(b)** a-b is a negative integer.
  - (c)  $\frac{b-a}{b+a}$ , the numerator and denominator are both positive, but the numerator is less than the denominator, so the answer is a positive rational number than is less than 1.
- **32.** If a and b are positive integers, then
  - (a) a + b is a positive integer
  - **(b)** a/b is a positive rational number
  - (c)  $a \times b$  is a positive integer
- 33. (a) Is the absolute value of a positive or a negative integer always an integer? |x| = x, so the absolute value of a positive integer is an integer. |-x| = x, so the absolute value of a negative integer is an integer.
  - **(b)** Is the reciprocal of a positive or negative integer always a rational number?

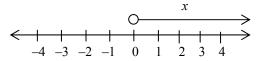
If x is a positive or negative integer, then the reciprocal of x is  $\frac{1}{x}$ . Since both 1 and x are integers, the reciprocal is a rational number.

- 34. (a) Is the absolute value of a positive or negative rational number rational? |x| = x, so if x is a positive or negative rational number, the absolute value of it is also a rational number.
  - **(b)** Is the reciprocal of a positive or negative rational number a rational number?

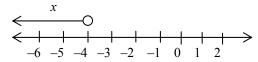
A rational number is a number that can be expressed as a fraction where both the numerator and denominator are integers and the denominator is not zero. So a rational number  $\frac{\text{integer }a}{\text{integer }b}$  has a reciprocal of

 $\frac{1}{\frac{\text{integer } a}{\text{integer } b}} = \frac{\text{integer } b}{\text{integer } a}, \text{ which is also a rational number if integer } a \text{ is not zero.}$ 

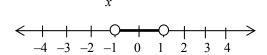
**35.** (a) If x > 0, then x is a positive number located to the right of zero on the number line.



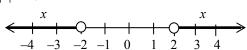
**(b)** If x < -4, then x is a negative number located to the left of -4 on the number line.



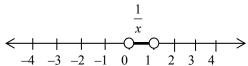
**36.** (a) If |x| < 1, then -1 < x < 1.



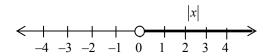
**(b)** |x| > 2, then x < -2 or x > 2



37. If x > 1, then  $\frac{1}{x}$  is a positive number less than 1. Or  $0 < \frac{1}{x} < 1$ .



**38.** If x < 0, then |x| is a positive number greater than zero.



**39.**  $a+bj=a+b\sqrt{-1}$  is a real number when  $\sqrt{-1}$  is eliminated, which is when b=0. So a+bj is a real number for all real values of a and b=0.

**40.** The variables are w and t.

The constants are c, 0.1, and 1.

**41.**  $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$ . Find  $C_T$ , where  $C_1 = 0.0040 \,\mathrm{F}$  and  $C_2 = 0.0010 \,\mathrm{F}$ .

$$\frac{1}{C_T} = \frac{1}{0.0040} + \frac{1}{0.0010}$$

$$\frac{1}{C_T} = \frac{1(0.0040) + 1(0.0010)}{0.0040 \times 0.0010}$$

$$C_T = \frac{0.0040 \times 0.0010}{0.0040 + 0.0010} = \frac{0.0000040}{0.0050}$$

$$C_T = 0.00080 \text{ F}$$

**42.** |100V| = 100V

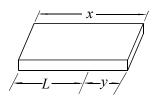
$$|-200V| = 200V$$

$$|-200V| > |100V|$$

43.  $N = \frac{a \text{ bits}}{\text{bytes}} \times \frac{1000 \text{ bytes}}{1 \text{ kilobyte}} \times n \text{ kilobytes}$ 

$$N = 1000 \ an \ bits$$

44.



$$x =$$
length of base in m

$$y =$$
 the shortened length in centimetres.

$$100x = \text{length of base in cm}$$

$$y + L = 100x$$
, all dimensions in cm

$$L = 100x - y$$

**45.** Yes,  $-20 \,^{\circ}\text{C} > -30 \,^{\circ}\text{C}$  because  $-30 \,^{\circ}\text{C}$  is found to the left of  $-20 \,^{\circ}\text{C}$  on the number line.

**46.** For I < 4 A,  $R > 12 \Omega$ .

#### 1.2 Fundamental Operations of Algebra

1. 
$$16-2\times(-2)=16-(-4)=16+4=20$$

2. 
$$\frac{-18}{-6} + 5 - (-2)(3) = 3 + 5 - (-6) = 8 + 6 = 14$$

3. 
$$\frac{-12}{8-2} + \frac{5-1}{2(-1)} = \frac{-12}{6} + \frac{4}{-2} = -2 + (-2) = -4$$

4. 
$$\frac{7 \times 6}{0 \times 0} = \frac{42}{0} = \text{is undefined}$$
, not indeterminate.

5. 
$$5+(-8)=5-8=-3$$

6. 
$$-4+(-7)=-4-7=-11$$

7. 
$$-3+9=6$$
 or alternatively  $-3+9=+(9-3)=+(6)=6$ 

8. 
$$18-21=-3$$
 or alternatively  $18-21=-(21-18)=-(3)=-3$ 

9. 
$$-19-(-16)=-19+16=-3$$

10. 
$$-8-(-10) = -8+10 = 2$$

11. 
$$7(-4) = -(7 \times 4) = -28$$

12. 
$$-9(3) = -27$$

13. 
$$-7(-5) = +(7 \times 5) = 35$$

14. 
$$\frac{-9}{3} = -3$$

15. 
$$\frac{-6(20-10)}{-3} = \frac{-6(10)}{-3} = \frac{-60}{-3} = 20$$

**16.** 
$$\frac{-28}{-7(5-6)} = \frac{-28}{-7(-1)} = \frac{-28}{7} = -4$$

17. 
$$-2(4)(-5) = -8(-5) = 40$$

**18.** 
$$-3(-4)(-6)=12(-6)=-72$$

19. 
$$2(2-7) \div 10 = 2(-5) \div 10 = -10 \div 10 = -1$$

**20.** 
$$\frac{-64}{-2|4-8|} = \frac{-64}{-2|-4|} = \frac{-64}{-2(4)} = \frac{-64}{-8} = 8$$

**21.** 
$$16 \div 2(-4) = 8(-4) = -32$$

**22.** 
$$-20 \div 5(-4) = -4(-4) = 16$$

**23.** 
$$-9 - |2 - 10| = -9 - |-8| = -9 - 8 = -17$$

**24.** 
$$(7-7) \div (5-7) = 0 \div (-2) = 0$$

**25.** 
$$\frac{17-7}{7-7} = \frac{10}{0}$$
 is undefined

**26.** 
$$\frac{(7-7)(2)}{(7-7)(-1)} = \frac{0(2)}{0(-1)} = \frac{0}{0}$$
 is indeterminate

**27.** 
$$8-3(-4)=8+12=20$$

**28.** 
$$-20+8 \div 4 = -20+2 = -18$$

**29.** 
$$-2(-6) + \left| \frac{8}{-2} \right| = 12 + \left| -4 \right| = 12 + 4 = 16$$

**30.** 
$$\frac{|-2|}{-2} = \frac{2}{-2} = -1$$

31. 
$$10(-8)(-3) \div (10-50) = 10(-8)(-3) \div (-40)$$
  
=  $-80(-3) \div (-40)$   
=  $240 \div (-40)$   
=  $-6$ 

32. 
$$\frac{7-|-5|}{-1(-2)} = \frac{7-5}{2} = \frac{2}{2} = 1$$

33. 
$$\frac{24}{3+(-5)}-4(-9)=\frac{24}{-2}+(4\times 9)=-12+36=24$$

34. 
$$\frac{-18}{3} - \frac{4 - |-6|}{-1} = \frac{-18}{3} - \frac{4 - 6}{-1} = -6 - \frac{-2}{-1} = -6 - 2 = -8$$

35. 
$$-7 - \frac{|-14|}{2(2-3)} - 3|6 - 8| = -7 - \frac{14}{2(-1)} - 3|-2|$$

$$= -7 - \frac{14}{-2} - 3(2)$$

$$= -7 - (-7) - 6$$

$$= -7 + 7 - 6$$

$$= -6$$

36. 
$$-7(-3) + \frac{-6}{-3} - |-9| = +(7 \times 3) + 2 - 9$$
  
=  $21 + 2 - 9$   
=  $14$ 

37. 
$$\frac{3|-9-2(-3)|}{1-10} = \frac{3|-9+6|}{-9}$$
$$= \frac{3|-3|}{-9}$$
$$= \frac{9}{-9}$$
$$= -1$$

38. 
$$\frac{20(-12)-40(-15)}{98-|-98|} = \frac{-240+600}{98-98} = \frac{360}{0} = \text{is undefined}$$

- **39.** 6(7) = (7)6 demonstrates the commutative law of multiplication.
- **40.** 6+8=8+6 demonstrates the commutative law of addition.
- **41.** 6(3+1) = 6(3) + 6(1) demonstrates the distributive law.
- **42.**  $4(5 \times \pi) = (4 \times 5)\pi$  demonstrates the associative law of multiplication.
- **43.** 3+(5+9)=(3+5)+9 demonstrates the associative law of addition.
- 44. 8(3-2) = 8(3) 8(2) demonstrates the distributive law.
- **45.**  $(\sqrt{5} \times 3) \times 9 = \sqrt{5} \times (3 \times 9)$  demonstrates the associative law of multiplication.
- **46.**  $(3\times6)\times7=7\times(3\times6)$  demonstrates the commutative law of multiplication.
- 47. -a+(-b)=-a-b, which is expression (d).
- **48.** b-(-a)=b+a=a+b, which is expression (a).
- **49.** -b (-a) = -b + a = a b, which is expression (b).

- **50.** -a (-b) = -a + b = b a, which is expression (c).
- **51.** Since |5-(-2)| = |5+2| = |7| = 7 and |-5-(-2)| = |-5+2| = |-3| = 3, |5-(-2)| > |-5-(-2)|.
- **52.** Since |-3-|-7| = |-3-7| = |-10| = 10 and ||-3|-7| = |3-7| = |-4| = 4, |-3-|-7| > ||-3|-7|.
- 53. (a) The sign of a product of an even number of negative numbers is positive. Example: -3(-6) = 18
  - (b) The sign of a product of an odd number of negative numbers is negative. Example: -5(-4)(-2) = -40
- **54.** Subtraction is not commutative because  $x y \neq y x$ . Example: 7 5 = 2 does not equal 5 7 = -2
- 55. Yes, from the definition in Section 1.1, the absolute value of a positive number is the number itself, and the absolute value of a negative number is the corresponding positive number. So for values of x where x > 0 (positive) or x = 0 (neutral) then |x| = x.

Example: |4| = 4.

The claim that absolute values of negative numbers |x| = -x is also true.

Example: if x is -6, then |-6| = -(-6) = 6.

**56.** The incorrect answer was achieved by subtracting before multiplying or dividing which violates the order of operations.  $24 - 6 \div 2 \times 3 \neq 18 \div 2 \times 3 = 9 \times 3 = 27$ 

The correct value is:

$$24-6 \div 2 \times 3 = 24-3 \times 3 = 24-9 = 15$$

- 57. (a) -xy = 1 is true for values of x and y that are negative reciprocals of each other or  $y = -\frac{1}{x}$ , providing that the number x in the denominator is not zero. So if x = 12, then  $y = -\frac{1}{12}$  and  $-xy = -(12)\left(-\frac{1}{12}\right) = 1$ .
  - **(b)**  $\frac{x-y}{x-y} = 1$  is true for all values of x and y, providing that  $x \neq y$  to prevent division by zero.
- 58. (a) |x+y| = |x| + |y| is true for values where both x and y have the same sign or either are zero:

$$|x+y| = |x| + |y|$$
, when  $x \ge 0$  and  $y \ge 0$  or when  $x \le 0$  and  $y \le 0$ 

Example:

$$|6+3| = 6+3 = 9$$
 and

$$|6| + |3| = 6 + 3 = 9$$

Also,

$$\left| -6 + (-3) \right| = \left| -9 \right| = 9$$

$$\left| -6 \right| + \left| -3 \right| = 6 + 3 = 9$$

|x+y| = |x| + |y| is not true however, when x and y have opposite signs

$$|x+y| \neq |x| + |y|$$
, when  $x > 0$  and  $y < 0$ ; or  $x < 0$  and  $y > 0$ .

$$|-21+6| = |-15| = 15,$$
  
 $|-21|+|6| = 21+6 = 27 \neq 15$ 

$$|4+(-5)| = |-1| = 1,$$
  
 $|4|+|-5| = 4+5 = 9 \ne 1$ 

(b) In order for |x-y|=|x|+|y| it is necessary that they have opposite signs or either to be zero. Symbolically, |x-y|=|x|+|y| when  $x \ge 0$  and  $y \le 0$ ; or when  $x \le 0$  and  $y \ge 0$ .

$$|6-(-3)| = 6+3=9$$
 and  $|6|+|-3| = 6+3=9$ 

#### Example:

$$|-11-7| = |-18| = 18$$
  
 $|-11| + |-7| = 11 + 7 = 18$ 

|x-y| = |x| + |y| is not true, however, when x and y have the same signs.

$$|x-y| \neq |x|+|y|$$
, when  $x > 0$  and  $y > 0$ ; or  $x < 0$  and  $y < 0$ .

#### Example:

$$|21-6| = |15| = 15,$$

$$|21| + |6| = 27 \neq 15$$

- **59.** The total change in the price of the stock is -0.68 + 0.42 + 0.06 + (-0.11) + 0.02 = -0.29.
- **60.** The difference in altitude is -86 (-1396) = 1396 86 = 1310 m
- **61.** The change in the meter energy reading *E* would be:

$$E_{change} = E_{used} - E_{generated}$$

$$E_{change} = 2.1 \text{ kW} \cdot \text{h} - 1.5 \text{ kW} (3.0 \text{ h})$$

$$E_{change} = 2.1 \text{ kW} \cdot \text{h} - 4.5 \text{ kW} \cdot \text{h}$$

$$E_{change} = -2.4 \text{ kW} \cdot \text{h}$$

- Assuming that this batting average is for the current season only which is just starting, the number of hits is zero and the total number of at-bats is also zero giving us a batting average =  $\frac{\text{number of hits}}{\text{at-bats}} = \frac{0}{0}$  which is indeterminate, not 0.000.
- **63.** The average temperature for the week is:

$$T_{\text{avg}} = \frac{-7 + (-3) + 2 + 3 + 1 + (-4) + (-6)}{7} \circ C$$

$$T_{\text{avg}} = \frac{-7 - 3 + 2 + 3 + 1 - 4 - 6}{7} \circ C$$

$$T_{\text{avg}} = \frac{-14}{7} \circ C = -2.0 \circ C$$

**64.** The vertical distance from the flare gun is

$$d = (70)(5) + (-16)(25)$$

$$d = 350 + (-400)$$

$$d = 350 - 400$$

$$d = -50 \text{ m}$$

The flare is 50 m below the flare gun.

**65.** The sum of the voltages is

$$V_{sum} = 6V + (-2V) + 8V + (-5V) + 3V$$

$$V_{sum} = 6V - 2V + 8V - 5V + 3V$$

$$V_{sum} = 10V$$

- **66.** (a) The change in the current for the first interval is the second reading the first reading  $Change_1 = -2 \text{ lb/in}^2 7 \text{ lb/in}^2 = -9 \text{ lb/in}^2$ .
  - **(b)** The change in the current for the middle intervals is the third reading the second reading  $Change_2 = -9 \text{ lb/in}^2 (-2 \text{ lb/in}^2) = -9 \text{ lb/in}^2 + 2 \text{ lb/in}^2 = -7 \text{ lb/in}^2$ .
  - (c) The change in the current for the last interval is the last reading the third reading  $Change_3 = -6 \text{ lb/in}^2 (-9 \text{ lb/in}^2) = -6 \text{ lb/in}^2 + 9 \text{ lb/in}^2 = 3 \text{ lb/in}^2$ .
- 67. The oil drilled by the first well is 100 m + 200 m = 300 m which equals the depth drilled by the second well 200 m + 100 m = 300 m.

100 m + 200 m = 200 m + 100 m demonstrates the commutative law of addition.

**68.** The first tank leaks  $12\frac{L}{h}(7 \text{ h}) = 84 \text{ L}$ . The second tank leaks  $7\frac{L}{h}(12 \text{h}) = 84 \text{L}$ .

 $12 \times 7 = 7 \times 12$  demonstrates the commutative law of multiplication.

69. The total time spent browsing these websites is the total time spent browsing the first site on each day + the total time spent browsing the second site on each day

$$t = 7 \text{ days} \times 25 \frac{\text{minutes}}{\text{day}} + 7 \text{ days} \times 15 \frac{\text{minutes}}{\text{day}}$$

$$t = 175 \text{ min} + 105 \text{ min}$$

$$t = 280 \text{ min}$$

OR

$$t = 7 \text{ days} \times (25 + 15) \frac{\text{minutes}}{\text{day}}$$

$$t = 7 \text{ days} \times 40 \frac{\text{minutes}}{\text{day}}$$

$$t = 280 \text{ min}$$

which illustrates the distributive law.

**70.** Distance = rate 
$$\times$$
 time

$$d = 600 \frac{\text{km}}{\text{h}} + 50 \frac{\text{km}}{\text{h}} 3 \text{ h}$$

$$d = 600 \frac{\text{km}}{\text{h}} (3\text{h}) + 50 \frac{\text{km}}{\text{h}} (3\text{h})$$

$$d = 1800 \text{ km} + 150 \text{ km} = 1950 \text{ km}$$

OR

$$d = 600 \frac{\text{km}}{\text{h}} + 50 \frac{\text{km}}{\text{h}} 3 \text{ h}$$

$$d = 650 \frac{\text{km}}{\text{h}} 3 \text{ h}$$

$$d = 1950 \text{ km}$$

This illustrates the distributive law.

#### 1.3 Calculators and Approximate Numbers

- 1. 0.390 has three significant digits since the zero is after the decimal. The zero is not necessary as a placeholder and should not be written unless it is significant.
- **2.** 35.303 rounded off to four significant digits is 35.30.
- 3. In finding the product of the approximate numbers,  $2.5 \times 30.5 = 76.25$ , but since 2.5 has 2 significant digits, the answer is 76.
- 4. 38.3 21.9(-3.58) = 116.702 using exact numbers; if we estimate the result, 40 20(-4) = 120.
- 5. 8 cylinders is exact because they can be counted. 55 km/h is approximate since it is measured.
- **6.** 0.002 mm thick is a measurement and is therefore an approximation. \$7.50 is an exact price.
- 7. 24 hr and 1440 min (60 min/h  $\times$  24 h = 1140 min) are both exact numbers.
- 8. 50 keys is exact because you can count them; 50 h of use is approximate since it is a measurement of time.
- **9.** Both 1 cm and 9 g are measured quantities and so they are approximate.
- 10. The numbers 90 and 75 are exact counts of windows while 15 years is a measurement of time, hence it is approximate.
- 11. 107 has 3 significant digits; 3004 has 4 significant digits; 1040 has 3 significant digits (the final zero is a placeholder.)
- 12. 3600 has 2 significant digits; 730 has 2 significant digits; 2055 has 4 significant digits.
- 13. 6.80 has 3 significant digits since the zero indicates precision; 6.08 has 3 significant digits; 0.068 has 2 significant digits (the zeros are placeholders.)
- 14. 0.8730 has 4 significant digits; 0.0075 has 2 significant digits; 0.0305 has 3 significant digits.
- 15. 3000 has 1 significant digit; 3000.1 has 5 significant digits; 3000.10 has 6 significant digits.

- 16. 1.00 has 3 significant digits since the zeros indicate precision; 0.01 has 1 significant digit since leading zeros are not significant; 0.0100 has 3 significant digits, counting the trailing zeros.
- 17. 5000 has 1 significant digit; 5000.0 has 5 significant digits; 5000 has 4 significant digits since the bar over the final zero indicates that it is significant.
- 18. 200 has 1 significant digit;  $20\overline{0}$  has 3 significant digits; 200.00 has 5 significant digits.
- **19.** (a) 0.010 has more decimal places (3) and is more precise.
  - **(b)** 30.8 has more significant digits (3) and is more accurate.
- 20. (a) Both 0.041 and 7.673 have the same precision as they have the same number of decimal places (3).
  - (b) 7.673 is more accurate because it has more significant digits (4) than 0.041, which has 2 significant digits.
- 21. (a) Both 0.1 and 78.0 have the same precision as they have the same number of decimal places.
  - (b) 78.0 is more accurate because it has more significant digits (3) than 0.1, which has 1 significant digit.
- 22. (a) 0.004 is more precise because it has more decimal places (3).
  - (b) 7040 is more accurate because it has more significant digits (3) than 0.004, which has only 1 significant digit.
- 23. (a) 0.004 is more precise because it has more decimal places (3).
  - **(b)** Both have the same accuracy as they both have 1 significant digit.
- **24.** The precision and accuracy of |-8.914| and 8.914 are the same.
  - (a) Both 50.060 and 8.914 have the same precision as they have the same number of decimal places (3).
  - (b) 50.060 is more accurate because it has more significant digits (5) than 8.914, which has 4 significant digits.
- **25.** (a) 4.936 rounded to 3 significant digits is 4.94.
  - **(b)** 4.936 rounded to 2 significant digits is 4.9.
- **26.** (a) 80.53 rounded to 3 significant digits is 80.5.
  - **(b)** 80.53 rounded to 2 significant digits is 81.
- **27.** (a) -50.893 rounded to 3 significant digits is -50.9.
  - **(b)** -50.893 rounded to 2 significant digits is -51.
- **28.** (a) 7.004 rounded to 3 significant digits is 7.00.
  - **(b)** 7.004 rounded to 2 significant digits is 7.0.
- **29.** (a) 5968 rounded to 3 significant digits is 5970.
  - **(b)** 5968 rounded to 2 significant digits is 6000.
- **30.** (a) 30.96 rounded to 3 significant digits is 31.0.
  - **(b)** 30.96 rounded to 2 significant digits is 31.
- **31.** (a) 0.9449 rounded to 3 significant digits is 0.945.
  - **(b)** 0.9449 rounded to 2 significant digits is 0.94.
- **32.** (a) 0.9999 rounded to 3 significant digits is 1.00.
  - **(b)** 0.9999 rounded to 2 significant digits is 1.0.
- 33. (a) Estimate: 13+1-2=12
  - **(b)** Calculator: 12.78 + 1.0495 1.633 = 12.1965, which is 12.20 to 0.01 precision

- **34.** (a) Estimate:  $4 \times 17 = 68$ 
  - **(b)** Calculator: 3.64(17.06) = 62.0984, which is 62.1 to 3 significant digits
- 35. (a) Estimate  $0.7 \times 4 9 = -6$ 
  - **(b)** Calculator:  $0.6572 \times 3.94 8.651 = -6.061632$ , which is -6.06 to 3 significant digits
- **36.** (a) Estimate  $40 26 \div 4 = 40 6.5 = 34$ 
  - **(b)** Calculator:  $41.5 26.4 \div 3.7 = 34.3648649$ , which is 34 to 2 significant digits
- 37. (a) Estimate 9+(1)(4)=9+4=13
  - **(b)** Calculator: 8.75 + (1.2)(3.84) = 13.358, which is 13 to 2 significant digits
- **38.** (a) Estimate  $30 \frac{20}{2} = 30 10 = 20$ 
  - **(b)** Calculator:  $28 \frac{20.955}{2.2} = 18.475$ , which is 18 to 2 significant digits
- **39.** (a) Estimate  $\frac{9(15)}{9+15} = \frac{135}{24} = 6$ , to 1 significant digit
  - **(b)** Calculator:  $\frac{8.75(15.32)}{8.75+15.32} = 5.569173$ , which is 5.57 to 3 significant digits
- **40.** (a) Estimate  $\frac{9(4)}{2+5} = \frac{36}{7} = 5$ , to 1 significant digit
  - **(b)** Calculator:  $\frac{8.97(4.003)}{2.0+4.78} = 5.296$ , which is 5.3 to 2 significant digits
- **41.** (a) Estimate  $4.5 \frac{2(300)}{400} = 3.0$ , to 2 significant digits
  - **(b)** Calculator:  $4.52 \frac{2.056(309.6)}{395.2} = 2.9093279$ , which is 2.91 to 3 significant digits
- 42. (a) Estimate  $8 + \frac{15}{2+2} = 12$ , to 2 significant digits
  - **(b)** Calculator:  $8.195 + \frac{14.9}{1.7 + 2.1} = 12.1160526$ , which is 12 to 2 significant digits
- 43. 0.9788+14.9=15.8788 since the least precise number in the question has 4 decimal places.
- 44. 17.311-22.98 = -5.669 since the least precise number in the question has 3 decimal places.
- **45.** -3.142(65) = -204.23, which is -204.2 because the least accurate number has 4 significant digits.
- **46.**  $8.62 \div 1728 = 0.004988$ , which is 0.00499 because the least accurate number has 3 significant digits.
- **47.** With a frequency listed as 2.75 MHz, the least possible frequency is 2.745 MHz, and the greatest possible frequency is 2.755 MHz. Any measurements between those limits would round to 2.75 MHz.
- **48.** For an engine displacement stated at 2400 cm<sup>3</sup>, the least possible displacement is 2350 cm<sup>3</sup>, and the greatest possible displacement is 2450 cm<sup>3</sup>. Any measurements between those limits would round to 2400 cm<sup>3</sup>.

- 49. The speed of sound is  $3.25 \text{ mi} \div 15 \text{ s} = 0.21666... \text{ mi/s} = 1144.0... \text{ ft/s}$ . However, the least accurate measurement was time since it has only 2 significant digits. The correct answer is 1100 ft/s.
- **50.** 4.4 s 2.72 s = 1.68 s, but the answer must be given according to precision of the least precise measurement in the question, so the correct answer is 1.7 s.
- **51.** (a)  $2.2+3.8\times4.5=2.2+(3.8\times4.5)=19.3$ 
  - **(b)**  $(2.2+3.8)\times4.5=6.0\times4.5=27$
- **52.** (a)  $6.03 \div 2.25 + 1.77 = (6.03 \div 2.25) + 1.77 = 4.45$ 
  - **(b)**  $6.03 \div (2.25 + 1.77) = 6.03 \div 4.02 = 1.5$
- 53. (a) 2+0=2
  - **(b)** 2-0=2
  - (c) 0-2=-2
  - (d)  $2 \times 0 = 0$
  - (e)  $2 \div 0 = \text{error}$ ; from Section 1.2, an equation that has 0 in the denominator is undefined when the numerator is not also 0.
- **54.** (a)  $2 \div 0.0001 = 20\ 000$ ;  $2 \div 0 = \text{error}$ 
  - **(b)**  $0.0001 \div 0.0001 = 1 ; 0 \div 0 = \text{error}$
  - (c) Any number divided by zero is undefined. Zero divided by zero is indeterminate.
- **55.**  $\pi = 3.14159265...$ 
  - (a)  $\pi < 3.1416$
  - **(b)**  $22 \div 7 = 3.1428$ 
    - $\pi\!<\!(22\div7)$
- **56.** (a)  $8 \div 33 = 0.2424... = 0.\overline{24}$ 
  - **(b)**  $\pi = 3.14159265...$
- 57. (a)  $1 \div 3 = 0.333...$  It is a rational number since it is a repeating decimal.
  - **(b)**  $5 \div 11 = 0.454545...$  It is a rational number since it is a repeating decimal.
  - (c)  $2 \div 5 = 0.400...$  It is a rational number since it is a repeating decimal (0 is the repeating part).
- 58.  $124 \div 990 = 0.12525...$  the calculator may show the answer as 0.1252525253 because it has rounded up for the next 5 that doesn't fit on the screen.
- 59. 32.4 MJ + 26.704 MJ + 36.23 MJ = 95.334 MJ. The answer must be to the same precision as the least precise measurement. The answer is 95.3 MJ.
- **60.** We would compute 8(68.6) + 5(15.3) = 625.3 and round to three significant digits for a total weight of 625 lb. The values 8 and 5 are exact.
- **61.** We would compute 12(129) + 16(298.8) = 6328.8 and round to three significant digits for a total weight of 6330 g. The values 12 and 16 are exact.
- **62.**  $V = (15.2 \Omega + 5.64 \Omega + 101.23 \Omega) \times 3.55 A$ 
  - $V = 122.07 \ \Omega \times 3.55 \ A$
  - V = 433.3485 V
  - V = 433 V to 3 significant digits