C H A P T E R 1
Preparation for Calculus

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Section 1.1 Graphs and Models

1. To find the x-intercepts of the graph of an equation, let y be zero and solve the equation for x. To find the y-intercepts of the graph of an equation, let x be zero and solve the equation for y.

2. Substitute the x- and y-values of the ordered pair into both equations. If the ordered pair satisfies both equations, then the ordered pair is a point of intersection.

3. $y = -\frac{3}{2}x + 3$

x-intercept: $(2, 0)$

y-intercept: $(0, 3)$

Matches graph (b).

4. $y = \sqrt{9 - x^2}$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: $(0, 3)$

Matches graph (d).

5. $y = 3 - x^2$

x-intercepts: $\left(\sqrt{3}, 0\right), \left(-\sqrt{3}, 0\right)$

y-intercept: $(0, 3)$

Matches graph (a).

6. $y = x^3 - x$

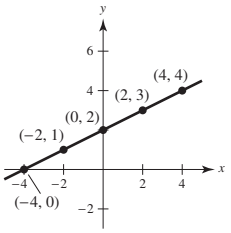
x-intercepts: $(0, 0), (-1, 0), (1, 0)$

y-intercept: $(0, 0)$

Matches graph (c).

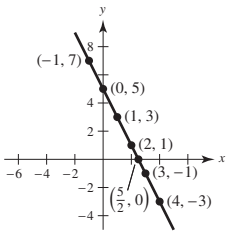
7. $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4



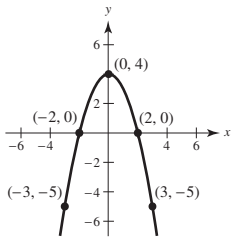
8. $y = 5 - 2x$

x	-1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	-1	-3



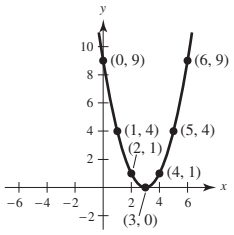
9. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



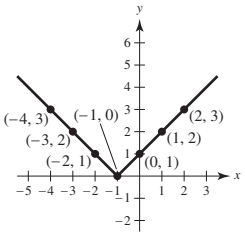
10. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



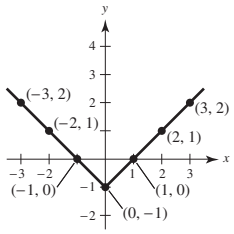
11. $y = |x + 1|$

x	-4	-3	-2	-1	0	1	2
y	3	2	1	0	1	2	3



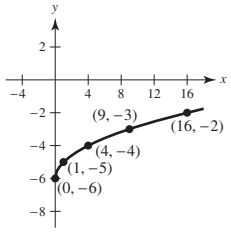
12. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



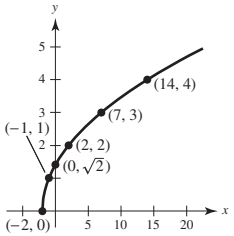
13. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



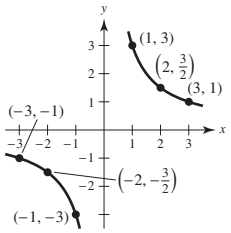
14. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



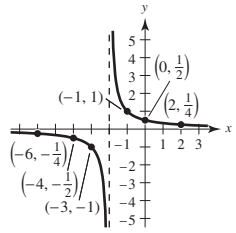
15. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1

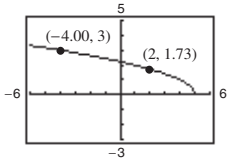


16. $y = \dfrac{1}{x + 2}$

x	-6	-4	-3	-2	-1	0	2
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



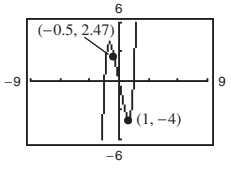
17. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

18. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

19. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5;\ (0, -5)$

x-intercept: $0 = 2x - 5$

$5 = 2x$

$x = \frac{5}{2};\ \left(\frac{5}{2}, 0\right)$

20. $y = 4x^2 + 3$

y-intercept: $y = 4(0)^2 + 3 = 3;\ (0, 3)$

x-intercept: $0 = 4x^2 + 3$

$-3 = 4x^2$

None. y cannot equal 0.

21. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2;\ (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1;\ (-2, 0), (1, 0)$

22. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0;\ (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2;\ (0, 0), (\pm 2, 0)$

23. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0;\ (0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4;\ (0, 0), (4, 0), (-4, 0)$

24. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1;\ (1, 0)$

25. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

y-intercept: $y = \dfrac{2 - \sqrt{0}}{5(0) + 1} = 2\ ;\ (0, 2)$

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x + 1}$

$0 = 2 - \sqrt{x}$

$x = 4\ ;\ (4, 0)$

26. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{[3(0) + 1]^2}$

$y = 0;\ (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3;\ (0, 0), (-3, 0)$

27. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0;\ (0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0;\ (0, 0)$

28. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = 2x - \sqrt{x^2 + 1}$

$2x = \sqrt{x^2 + 1}$

$4x^2 = x^2 + 1$

$3x^2 = 1$

$x^2 = \dfrac{1}{3}$

$x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\sqrt{3}}{3};\ \left(\dfrac{\sqrt{3}}{3}, 0\right)$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

29. Symmetric with respect to the y-axis because
$y = (-x)^2 - 6 = x^2 - 6$.

30. $y = 9x - x^2$

No symmetry with respect to either axis or the origin.

31. Symmetric with respect to the x-axis because
$(-y)^2 = y^2 = x^3 - 8x$.

32. Symmetric with respect to the origin because

$$(-y) = (-x)^3 + (-x)$$
$$-y = -x^3 - x$$
$$y = x^3 + x.$$

33. Symmetric with respect to the origin because
$(-x)(-y) = xy = 4$.

34. Symmetric with respect to the x-axis because
$x(-y)^2 = xy^2 = -10$.

35. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

36. Symmetric with respect to the origin because

$$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$$
$$xy - \sqrt{4 - x^2} = 0.$$

37. Symmetric with respect to the origin because

$$-y = \frac{-x}{(-x)^2 + 1}$$
$$y = \frac{x}{x^2 + 1}.$$

38. Symmetric with respect to the origin because

$$-y = \frac{(-x)^5}{4 - (-x)^2}$$
$$-y = \frac{-x^5}{4 - x^2}$$
$$y = \frac{x^5}{4 - x^2}.$$

39. $y = |x^3 + x|$ is symmetric with respect to the y-axis because $y = |(-x)^3 + (-x)| = |-(x^3 + x)| = |x^3 + x|$.

40. $|y| - x = 3$ is symmetric with respect to the x-axis because

$$|-y| - x = 3$$
$$|y| - x = 3.$$

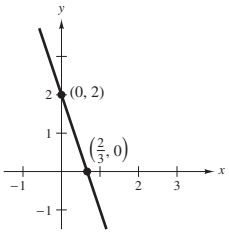
41. $y = 2 - 3x$

$y = 2 - 3(0) = 2$, y-intercept

$0 = 2 - 3(x) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$, x-intercept

Intercepts: $(0, 2)$, $\left(\frac{2}{3}, 0\right)$

Symmetry: none



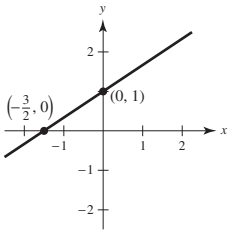
42. $y = \frac{2}{3}x + 1$

$y = \frac{2}{3}(0) + 1 = 1$, y-intercept

$0 = \frac{2}{3}x + 1 \Rightarrow -\frac{2}{3}x = 1 \Rightarrow x = -\frac{3}{2}$, x-intercept

Intercepts: $(0, 1)$, $\left(-\frac{3}{2}, 0\right)$

Symmetry: none



43. $y = 9 - x^2$

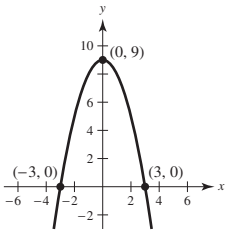
$y = 9 - (0)^2 = 9$, y-intercept

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$, x-intercepts

Intercepts: $(0, 9)$, $(3, 0)$, $(-3, 0)$

$y = 9 - (-x)^2 = 9 - x^2$

Symmetry: y-axis



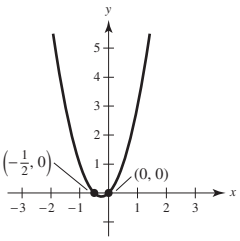
44. $y = 2x^2 + x = x(2x + 1)$

$y = 0(2(0) + 1) = 0$, y-intercept

$0 = x(2x + 1) \Rightarrow x = 0, -\frac{1}{2}$, x-intercepts

Intercepts: $(0, 0)$, $\left(-\frac{1}{2}, 0\right)$

Symmetry: none



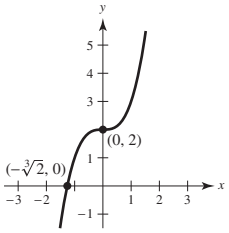
45. $y = x^3 + 2$

$y = 0^3 + 2 = 2$, y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2}$, x-intercept

Intercepts: $\left(-\sqrt[3]{2}, 0\right), (0, 2)$

Symmetry: none



46. $y = x^3 - 4x$

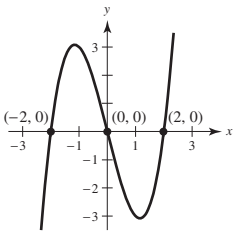
$y = 0^3 - 4(0) = 0$, y-intercept

$$x^3 - 4x = 0$$
$$x(x^2 - 4) = 0$$
$$x(x + 2)(x - 2) = 0$$
$$x = 0, \pm 2, \text{ } x\text{-intercepts}$$

Intercepts: $(0, 0), (2, 0), (-2, 0)$

$y = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$

Symmetry: origin



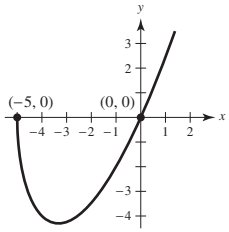
47. $y = x\sqrt{x + 5}$

$y = 0\sqrt{0 + 5} = 0$, y-intercept

$x\sqrt{x + 5} = 0 \Rightarrow x = 0, -5$, x-intercepts

Intercepts: $(0, 0), (-5, 0)$

Symmetry: none



48. $y = \sqrt{25 - x^2}$

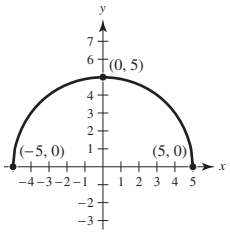
$y = \sqrt{25 - 0^2} = \sqrt{25} = 5$, y-intercept

$$\sqrt{25 - x^2} = 0$$
$$25 - x^2 = 0$$
$$(5 + x)(5 - x) = 0$$
$$x = \pm 5, \text{ } x\text{-intercept}$$

Intercepts: $(0, 5), (5, 0), (-5, 0)$

$y = \sqrt{25 - (-x)^2} = \sqrt{25 - x^2}$

Symmetry: y-axis



49. $x = y^3$

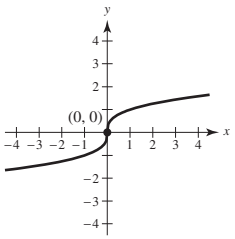
$y^3 = 0 \Rightarrow y = 0$, y-intercept

$x = 0$, x-intercept

Intercept: $(0, 0)$

$-x = (-y)^3 \Rightarrow -x = -y^3$

Symmetry: origin



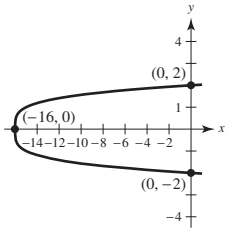
50. $x = y^4 - 16$

$$y^4 - 16 = 0$$
$$(y^2 - 4)(y^2 + 4) = 0$$
$$(y - 2)(y + 2)(y^2 + 4) = 0$$
$$y = \pm 2, \text{ } y\text{-intercepts}$$

$x = 0^4 - 16 = -16$, x-intercept

Intercepts: $(0, 2), (0, -2), (-16, 0)$

Symmetry: x-axis because $x = (-y)^4 - 16 = y^4 - 16$



51. $y = \dfrac{8}{x}$

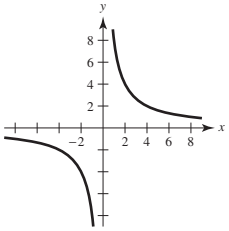
$y = \dfrac{8}{0} \Rightarrow$ Undefined $\Rightarrow$ no y-intercept

$\dfrac{8}{x} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercepts: none

$-y = \dfrac{8}{-x} \Rightarrow y = \dfrac{8}{x}$

Symmetry: origin



52. $y = \dfrac{10}{x^2 + 1}$

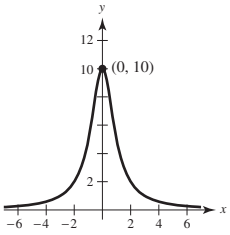
$y = \dfrac{10}{0^2 + 1} = 10$, y-intercept

$\dfrac{10}{x^2 + 1} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercepts

Intercept: (0, 10)

$y = \dfrac{10}{(-x)^2 + 1} = \dfrac{10}{x^2 + 1}$

Symmetry: y-axis



53. $y = 6 - |x|$

$y = 6 - |0| = 6$, y-intercept

$6 - |x| = 0$

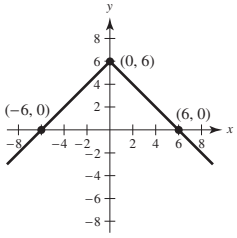
$6 = |x|$

$x = \pm 6$, x-intercepts

Intercepts: $(0, 6)$, $(-6, 0)$, $(6, 0)$

$y = 6 - |-x| = 6 - |x|$

Symmetry: y-axis



54. $y = |6 - x|$

$y = |6 - 0| = |6| = 6$, y-intercept

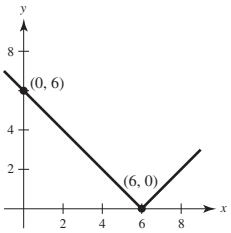
$|6 - x| = 0$

$6 - x = 0$

$6 = x$, x-intercept

Intercepts: (0, 6), (6, 0)

Symmetry: none



55. $3y^2 - x = 9$

$3y^2 = x + 9$

$y^2 = \frac{1}{3}x + 3$

$y = \pm\sqrt{\frac{1}{3}x + 3}$

$y = \pm\sqrt{0 + 3} = \pm\sqrt{3}$, y-intercepts

$\pm\sqrt{\frac{1}{3}x + 3} = 0$

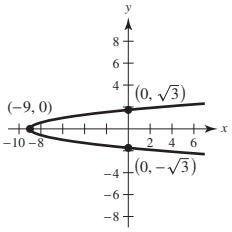
$\frac{1}{3}x + 3 = 0$

$x = -9$, x-intercept

Intercepts: $\left(0, \sqrt{3}\right)$, $\left(0, -\sqrt{3}\right)$, $(-9, 0)$

$3(-y)^2 - x = 3y^2 - x = 9$

Symmetry: x-axis



56. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

$y = \pm\dfrac{\sqrt{4 - 0^2}}{2} = \pm\dfrac{\sqrt{4}}{2} = \pm 1$, y-intercepts

$x^2 + 4(0)^2 = 4$

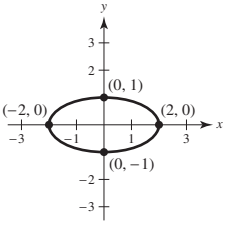
$x^2 = 4$

$x = \pm 2$, x-intercepts

Intercepts: $(-2, 0)$, $(2, 0)$, $(0, -1)$, $(0, 1)$

$(-x)^2 + 4(-y)^2 = 4 \Rightarrow x^2 + 4y^2 = 4$

Symmetry: origin and both axes



57. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$8 - x = 4x - 7$

$15 = 5x$

$3 = x$

The corresponding y-value is $y = 5$.

Point of intersection: $(3, 5)$

58. $3x - 2y = -4 \Rightarrow y = \dfrac{3x + 4}{2}$

$4x + 2y = -10 \Rightarrow y = \dfrac{-4x - 10}{2}$

$\dfrac{3x + 4}{2} = \dfrac{-4x - 10}{2}$

$3x + 4 = -4x - 10$

$7x = -14$

$x = -2$

The corresponding y-value is $y = -1$.

Point of intersection: $(-2, -1)$

59. $x^2 + y = 15 \Rightarrow y = -x^2 + 15$

$-3x + y = 11 \Rightarrow y = 3x + 11$

$-x^2 + 15 = 3x + 11$

$0 = x^2 + 3x - 4$

$0 = (x + 4)(x - 1)$

$x = -4, 1$

The corresponding y-values are $y = -1$ (for $x = -4$) and $y = 14$ (for $x = 1$).

Points of intersection: $(-4, -1), (1, 14)$

60. $x = 3 - y^2 \Rightarrow y^2 = 3 - x$

$y = x - 1$

$3 - x = (x - 1)^2$

$3 - x = x^2 - 2x + 1$

$0 = x^2 - x - 2 = (x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

61. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

$5 - x^2 = (x - 1)^2$

$5 - x^2 = x^2 - 2x + 1$

$0 = 2x^2 - 2x - 4 = 2(x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

62. $x^2 + y^2 = 16$

$x + 2y = 4 \quad \Rightarrow \quad x = 4 - 2y$

$(4 - 2y)^2 + y^2 = 16$

$5y^2 - 16y + 16 = 16$

$y(5y - 16) = 0 \quad \Rightarrow \quad y = 0, \frac{16}{5}$

$x = 4 - 2(0) \Rightarrow x = 4$

$x = 4 - 2\left(\frac{16}{5}\right) \Rightarrow x = -\frac{12}{5}$

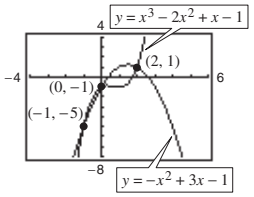
Points of intersection: $(4, 0), \left(-\frac{12}{5}, \frac{16}{5}\right)$

63. $y = x^3 - 2x^2 + x - 1$

$y = -x^2 + 3x - 1$

Points of intersection:

$(-1, -5), (0, -1), (2, 1)$



Analytically,

$x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$

$x^3 - x^2 - 2x = 0$

$x(x - 2)(x + 1) = 0$

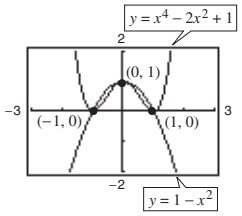
$x = -1, 0, 2.$

64. $y = x^4 - 2x^2 + 1$

$y = 1 - x^2$

Points of intersection:

$(-1, 0), (0, 1), (1, 0)$



Analytically,

$1 - x^2 = x^4 - 2x^2 + 1$

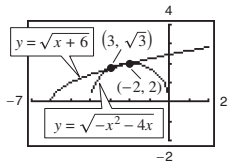
$0 = x^4 - x^2$

$0 = x^2(x + 1)(x - 1)$

$x = -1, 0, 1.$

65. $y = \sqrt{x+6}$

$y = \sqrt{-x^2 - 4x}$



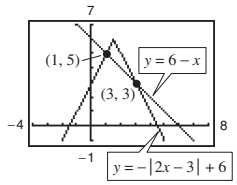
Points of intersection: $(-2, 2)$, $\left(-3, \sqrt{3}\right) \approx (-3, 1.732)$

Analytically, $\sqrt{x+6} = \sqrt{-x^2 - 4x}$

$$x + 6 = -x^2 - 4x$$
$$x^2 + 5x + 6 = 0$$
$$(x+3)(x+2) = 0$$
$$x = -3, -2.$$

66. $y = -|2x - 3| + 6$

$y = 6 - x$



Points of intersection: (3, 3), (1, 5)

Analytically, $-|2x - 3| + 6 = 6 - x$

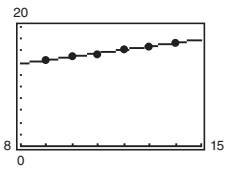
$$|2x - 3| = x$$

$2x - 3 = x$ or $2x - 3 = -x$

$x = 3$ or $x = 1$.

67. (a) Using a graphing utility, you obtain

$y = 0.58t + 9.2$.

(b) 

The model is a good fit for the data.

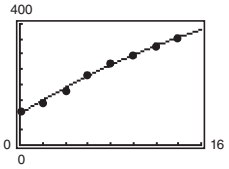
(c) For 2024, $t = 24$:

$y = 0.58(24) + 9.2 \approx 23.1$

The GDP in 2024 will be approximately \$23.1 trillion.

68. (a) Using a graphing utility, you obtain

$y = -0.35t^2 + 22.9t + 103$.

(b) 

The model is a good fit for the data.

(c) For 2024, $t = 24$:

$y = -0.35(24)^2 + 22.9(24) + 103 \approx 451$

There will be approximately 451 million cell phone subscribers in 2024.

69. $C = R$

$$2.04x + 5600 = 3.29x$$
$$5600 = 3.29x - 2.04x$$
$$5600 = 1.25x$$
$$x = \frac{5600}{1.25} = 4480$$

To break even, 4480 units must be sold.

70. $y^2 = 4kx$

(a) $(1, 1)$: $1^2 = 4k(1)$

$1 = 4k$

$k = \frac{1}{4}$

(b) $(2, 4)$: $(4)^2 = 4k(2)$

$16 = 8k$

$k = 2$

(c) $(0, 0)$: $0^2 = 4k(0)$

k can be any real number.

(d) $(3, 3)$: $(3)^2 = 4k(3)$

$9 = 12k$

$k = \frac{9}{12} = \frac{3}{4}$

71. Answers may vary. *Sample answer*:

$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at

$x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.