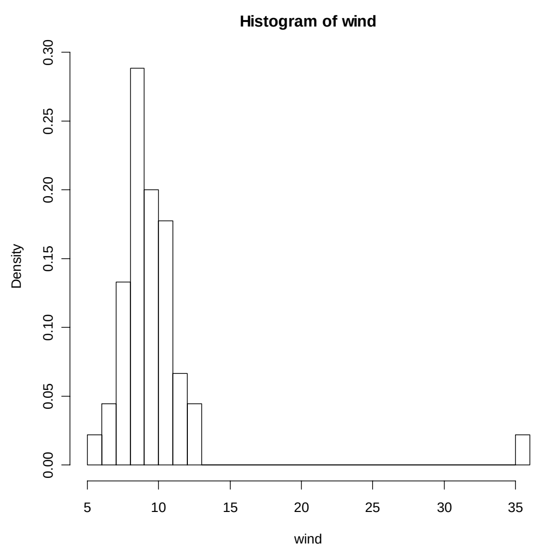
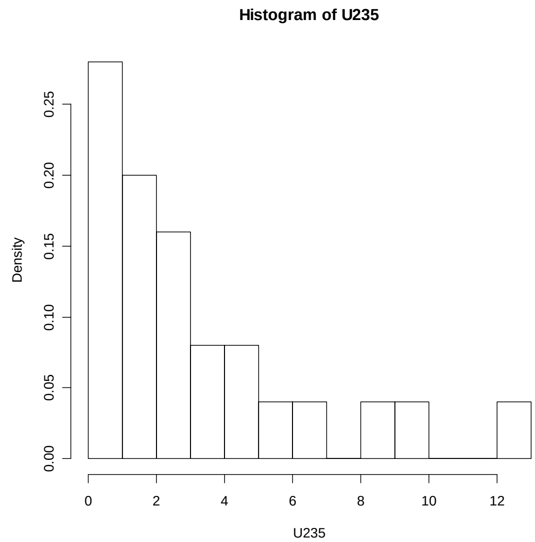
Chapter 1: What is Statistics?

1.1

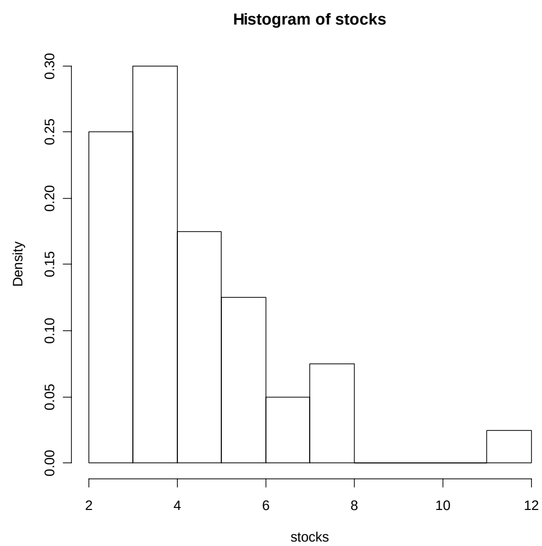
a. Population: all tires manufactured by the company for the specific year. Objective: to estimate the proportion of tires with unsafe tread.
b. Population: all adult residents of the particular state. Objective: to estimate the proportion who favor a unicameral legislature.
c. Population: times until recurrence for all people who have had a particular disease. Objective: to estimate the true average time until recurrence.
d. Population: lifetime measurements for all resistors of this type. Objective: to estimate the true mean lifetime (in hours).
e. Population: all generation X age US citizens (specifically, assign a '1' to those who want to start their own business and a '0' to those who do not, so that the population is the set of 1's and 0's). Objective: to estimate the proportion of generation X age US citizens who want to start their own business.
f. Population: all healthy adults in the US. Objective: to estimate the true mean body temperature
g. Population: single family dwelling units in the city. Objective: to estimate the true mean water consumption



1.2 **a.** This histogram is above.
b. Yes, it is quite windy there.
c. 11/45, or approx. 24.4%
d. it is not especially windy in the overall sample.



1.3 The histogram is above.

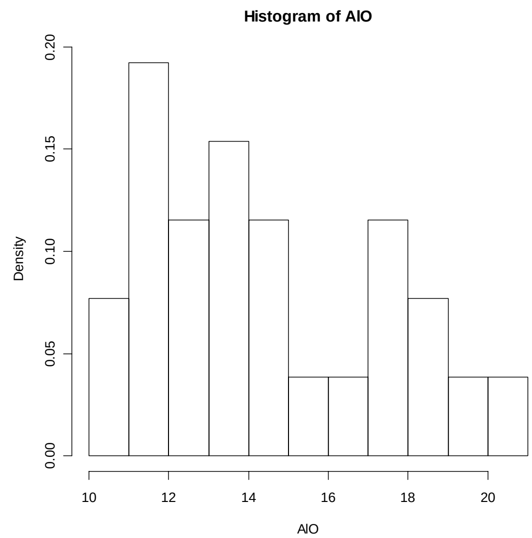


1.4 **a.** The histogram is above.
b. 18/40 = 45%
c. 29/40 = 72.5%

1.5 **a.** The categories with the largest grouping of students are 2.45 to 2.65 and 2.65 to 2.85. (both have 7 students).
b. 7/30
c. 7/30 + 3/30 + 3/30 + 3/30 = 16/30

1.6 **a.** The modal category is 2 (quarts of milk). About 36% (9 people) of the 25 are in this category.
b. .2 + .12 + .04 = .36
c. Note that 8% purchased 0 while 4% purchased 5. Thus, 1 – .08 – .04 = .88 purchased between 1 and 4 quarts.

1.7 **a.** There is a possibility of bimodality in the distribution.
b. There is a dip in heights at 68 inches.
c. If all of the students are roughly the same age, the bimodality could be a result of the men/women distributions.



1.8 **a.** The histogram is above.
b. The data appears to be bimodal. Llanederyn and Caldicot have lower sample values than the other two.

1.9 **a.** Note that 9.7 = 12 – 2.3 and 14.3 = 12 + 2.3. So, (9.7, 14.3) should contain approximately 68% of the values.
b. Note that 7.4 = 12 – 2(2.3) and 16.6 = 12 + 2(2.3). So, (7.4, 16.6) should contain approximately 95% of the values.
c. From parts (a) and (b) above, 95% - 68% = 27% lie in both (14.3. 16.6) and (7.4, 9.7). By symmetry, 13.5% should lie in (14.3, 16.6) so that 68% + 13.5% = 81.5% are in (9.7, 16.6)
d. Since 5.1 and 18.9 represent three standard deviations away from the mean, the proportion outside of these limits is approximately 0.

1.10 **a.** 14 – 17 = -3.
b. Since 68% lie within one standard deviation of the mean, 32% should lie outside. By symmetry, 16% should lie below one standard deviation from the mean.
c. If normally distributed, approximately 16% of people would spend less than –3 hours on the internet. Since this doesn't make sense, the population is not normal.

1.11 **a.** $\sum_{i=1}^{n} c = c + c + \ldots + c = nc$.

b. $\sum_{i=1}^{n} cy_i = c(y_1 + \ldots + y_n) = c\sum_{i=1}^{n} y_i$

c. $\sum_{i=1}^{n} (x_i + y_i) = x_1 + y_1 + x_2 + y_2 + \ldots + x_n + y_n = (x_1 + x_2 + \ldots + x_n) + (y_1 + y_2 + \ldots + y_n)$

Using the above, the numerator of s^2 is $\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}(y_i^2 - 2y_i\bar{y} + \bar{y}^2) = \sum_{i=1}^{n} y_i^2 -$ $2\bar{y}\sum_{i=1}^{n} y_i + n\bar{y}^2$ Since $n\bar{y} = \sum_{i=1}^{n} y_i$, we have $\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - n\bar{y}^2$. Let $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ to get the result.

1.12 Using the data, $\sum_{i=1}^{6} y_i = 14$ and $\sum_{i=1}^{6} y_i^2 = 40$. So, $s^2 = (40 - 14^2/6)/5 = 1.47$. So, $s = 1.21$.

1.13 **a.** With $\sum_{i=1}^{45} y_i = 440.6$ and $\sum_{i=1}^{45} y_i^2 = 5067.38$, we have that $\bar{y} = 9.79$ and $s = 4.14$.

b.

k	interval	frequency	Exp. frequency
1	5.65, 13.93	44	30.6
2	1.51, 18.07	44	42.75
3	-2.63, 22.21	44	45

1.14 **a.** With $\sum_{i=1}^{25} y_i = 80.63$ and $\sum_{i=1}^{25} y_i^2 = 500.7459$, we have that $\bar{y} = 3.23$ and $s = 3.17$.

b.

k	interval	frequency	Exp. frequency
1	0.063, 6.397	21	17
2	-3.104, 9.564	23	23.75
3	-6.271, 12.731	25	25

1.15 **a.** With $\sum_{i=1}^{40} y_i = 175.48$ and $\sum_{i=1}^{40} y_i^2 = 906.4118$, we have that $\bar{y} = 4.39$ and $s = 1.87$.

b.

k	interval	frequency	Exp. frequency
1	2.52, 6.26	35	27.2
2	0.65, 8.13	39	38
3	-1.22, 10	39	40

1.16 **a.** Without the extreme value, $\bar{y} = 4.19$ and $s = 1.44$.
b. These counts compare more favorably:

k	interval	frequency	Exp. frequency
1	2.75, 5.63	25	26.52
2	1.31, 7.07	36	37.05
3	-0.13, 8.51	39	39

1.17 For Ex. 1.2, range/4 = 7.35, while s = 4.14. For Ex. 1.3, range/4 = 3.04, while = s = 3.17. For Ex. 1.4, range/4 = 2.32, while s = 1.87.

1.18 The approximation is (800–200)/4 = 150.

1.19 One standard deviation below the mean is 34 – 53 = –19. The empirical rule suggests that 16% of all measurements should lie one standard deviation below the mean. Since chloroform measurements cannot be negative, this population cannot be normally distributed.

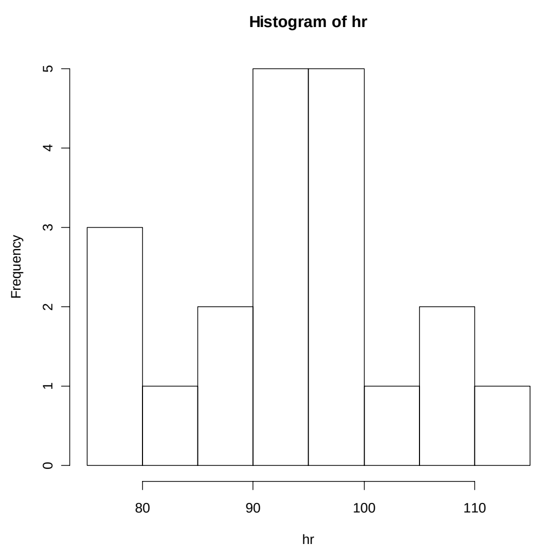
1.20 Since approximately 68% will fall between \$390 (\$420 – \$30) to \$450 (\$420 + \$30), the proportion above \$450 is approximately 16%.

1.21 (Similar to exercise 1.20) Having a gain of more than 20 pounds represents all measurements greater than one standard deviation below the mean. By the empirical rule, the proportion above this value is approximately 84%, so the manufacturer is probably correct.

1.22 (See exercise 1.11) $\sum_{i=1}^{n}(y_i - \overline{y}) = \sum_{i=1}^{n} y_i - n\overline{y} = \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} y_i = 0$.

1.23 **a.** (Similar to exercise 1.20) 95 sec = 1 standard deviation above 75 sec, so this percentage is 16% by the empirical rule.
b. (35 sec., 115 sec) represents an interval of 2 standard deviations about the mean, so approximately 95%
c. 2 minutes = 120 sec = 2.5 standard deviations above the mean. This is unlikely.

1.24 **a.** (112-78)/4 = 8.5



b. The histogram is above.

c. With $\sum_{i=1}^{20} y_i$ = 1874.0 and $\sum_{i=1}^{20} y_i^2$ = 117,328.0, we have that $\overline{y}$ = 93.7 and s = 9.55.

d.

k	interval	frequency	Exp. frequency
1	84.1, 103.2	13	13.6
2	74.6, 112.8	20	19
3	65.0, 122.4	20	20

1.25 **a.** (716-8)/4 = 177
b. The figure is omitted.
c. With $\sum_{i=1}^{88} y_i$ = 18,550 and $\sum_{i=1}^{88} y_i^2$ = 6,198,356, we have that $\bar{y}$ = 210.8 and s = 162.17.
d.

k	interval	frequency	Exp. frequency
1	48.6, 373	63	59.84
2	-113.5, 535.1	82	83.6
3	-275.7, 697.3	87	88

1.26 For Ex. 1.12, 3/1.21 = 2.48. For Ex. 1.24, 34/9.55 = 3.56. For Ex. 1.25, 708/162.17 = 4.37. The ratio increases as the sample size increases.

1.27 (64, 80) is one standard deviation about the mean, so 68% of 340 or approx. 231 scores. (56, 88) is two standard deviations about the mean, so 95% of 340 or 323 scores.

1.28 (Similar to 1.23) 13 mg/L is one standard deviation below the mean, so 16%.

1.29 If the empirical rule is assumed, approximately 95% of all bearing should lie in (2.98, 3.02) – this interval represents two standard deviations about the mean. So, approximately 5% will lie outside of this interval.

1.30 If $\mu = 0$ and $\sigma = 1.2$, we expect 34% to be between 0 and 0 + 1.2 = 1.2. Also, approximately 95%/2 = 47.5% will lie between 0 and 2.4. So, 47.5% – 34% = 13.5% should lie between 1.2 and 2.4.

1.31 Assuming normality, approximately 95% will lie between 40 and 80 (the standard deviation is 10). The percent below 40 is approximately 2.5% which is relatively unlikely.

1.32 For a sample of size n, let n' denote the number of measurements that fall outside the interval $\bar{y} \pm ks$, so that $(n - n')/n$ is the fraction that falls inside the interval. To show this fraction is greater than or equal to $1 - 1/k^2$, note that

$$(n-1)s^2 = \sum_{i\in A}(y_i - \bar{y})^2 + \sum_{i\in b}(y_i - \bar{y})^2 \text{, (both sums must be positive)}$$

where $A = \{i: |y_i - \bar{y}| \geq ks\}$ and $B = \{i: |y_i - \bar{y}| < ks\}$. We have that $\sum_{i\in A}(y_i - \bar{y})^2 \geq \sum_{i\in A} k^2 s^2 = n'k^2s^2$, since if i is in A, $|y_i - \bar{y}| \geq ks$ and there are n' elements in A. Thus, we have that $s^2 \geq k^2s^2n'/(n-1)$, or $1 \geq k^2n'/(n-1) \geq k^2n'/n$. Thus, $1/k^2 \geq n'/n$ or $(n - n')/n \geq 1 - 1/k^2$.

1.33 With k =2, at least 1 – 1/4 = 75% should lie within 2 standard deviations of the mean. The interval is (0.5, 10.5).

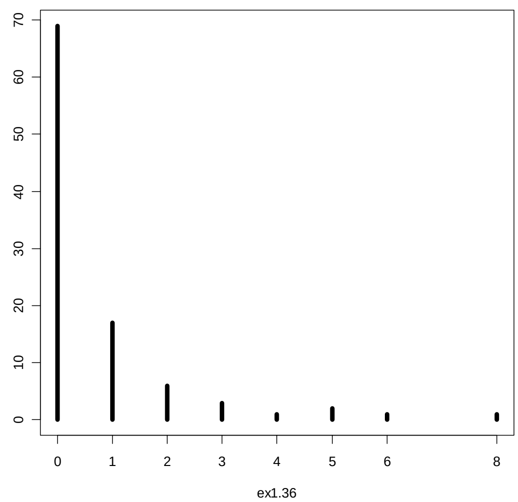
1.34 The point 13 is 13 – 5.5 = 7.5 units above the mean, or 7.5/2.5 = 3 standard deviations above the mean. By Tchebysheff's theorem, at least $1 - 1/3^2 = 8/9$ will lie within 3 standard deviations of the mean. Thus, at most 1/9 of the values will exceed 13.

1.35 **a.** (172 – 108)/4 =16

b. With $\sum_{i=1}^{15} y_i = 2041$ and $\sum_{i=1}^{15} y_i^2 = 281{,}807$ we have that $\bar{y} = 136.1$ and $s = 17.1$

c. a = 136.1 – 2(17.1) = 101.9, b = 136.1 + 2(17.1) = 170.3.

d. There are 14 observations contained in this interval, and 14/15 = 93.3%. 75% is a lower bound.



1.36 **a.** The histogram is above.

b. With $\sum_{i=1}^{100} y_i = 66$ and $\sum_{i=1}^{100} y_i^2 = 234$ we have that $\bar{y} = 0.66$ and $s = 1.39$.

c. Within two standard deviations: 95, within three standard deviations: 96. The calculations agree with Tchebysheff's theorem.

1.37 Since the lead readings must be non negative, 0 (the smallest possible value) is only 0.33 standard deviations from the mean. This indicates that the distribution is skewed.

1.38 By Tchebysheff's theorem, at least 3/4 = 75% lie between (0, 140), at least 8/9 lie between (0, 193), and at least 15/16 lie between (0, 246). The lower bounds are all truncated a 0 since the measurement cannot be negative.