Chapter R
Review

Section R.1

1. rational

2. $4+5\cdot 6-3=4+30-3=31$

3. Distributive

4. $5(x+3)=6$

5. True

6. False; The Zero-Product Property states that if a product equals 0, then at least one of the factors must equal 0.

7. False; 6 is the Greatest Common Factor of 12 and 18. The Least Common Multiple is the smallest value that both numbers will divide evenly. The LCM for 12 and 18 is 36.

8. True

9. $A\cup B=\{1,3,4,5,9\}\cup\{2,4,6,7,8\}$
$=\{1,2,3,4,5,6,7,8,9\}$

10. $A\cup C=\{1,3,4,5,9\}\cup\{1,3,4,6\}$
$=\{1,3,4,5,6,9\}$

11. $A\cap B=\{1,3,4,5,9\}\cap\{2,4,6,7,8\}=\{4\}$

12. $A\cap C=\{1,3,4,5,9\}\cap\{1,3,4,6\}=\{1,3,4\}$

13. $(A\cup B)\cap C$
$=(\{1,3,4,5,9\}\cup\{2,4,6,7,8\})\cap\{1,3,4,6\}$
$=\{1,2,3,4,5,6,7,8,9\}\cap\{1,3,4,6\}$
$=\{1,3,4,6\}$

14. $(A\cap B)\cup C$
$=(\{1,3,4,5,9\}\cap\{2,4,6,7,8\})\cup\{1,3,4,6\}$
$=\{4\}\cup\{1,3,4,6\}$
$=\{1,3,4,6\}$

15. $\overline{A}=\{0,2,6,7,8\}$

16. $\overline{C}=\{0,2,5,7,8,9\}$

17. $\overline{A\cap B}=\overline{\{1,3,4,5,9\}\cap\{2,4,6,7,8\}}$
$=\overline{\{4\}}=\{0,1,2,3,5,6,7,8,9\}$

18. $\overline{B\cup C}=\overline{\{2,4,6,7,8\}\cup\{1,3,4,6\}}$
$=\overline{\{1,2,3,4,6,7,8\}}=\{0,5,9\}$

19. $\overline{A}\cup\overline{B}=\{0,2,6,7,8\}\cup\{0,1,3,5,9\}$
$=\{0,1,2,3,5,6,7,8,9\}$

20. $\overline{B}\cap\overline{C}=\{0,1,3,5,9\}\cap\{0,2,5,7,8,9\}$
$=\{0,5,9\}$

21. **a.** $\{2,5\}$

b. $\{-6,2,5\}$

c. $\left\{-6,\frac{1}{2},-1.333...=-1.\overline{3},2,5\right\}$

d. $\{\pi\}$

e. $\left\{-6,\frac{1}{2},-1.333...=-1.\overline{3},\pi,2,5\right\}$

22. **a.** $\{1\}$

b. $\{0,1\}$

c. $\left\{-\frac{5}{3},2.060606...=2.\overline{06},1.25,0,1\right\}$

d. $\left\{\sqrt{5}\right\}$

e. $\left\{-\frac{5}{3},2.060606...=2.\overline{06},1.25,0,1,\sqrt{5}\right\}$

23. **a.** $\{1\}$

b. $\{0,1\}$

c. $\left\{0,1,\frac{1}{2},\frac{1}{3},\frac{1}{4}\right\}$

d. None

e. $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$

24. a. None

b. $\{-1\}$

c. $\{-1.3, -1.2, -1.1, -1\}$

d. None

e. $\{-1.3, -1.2, -1.1, -1\}$

25. a. None

b. None

c. None

d. $\left\{\sqrt{2}, \pi, \sqrt{2}+1, \pi+\frac{1}{2}\right\}$

e. $\left\{\sqrt{2}, \pi, \sqrt{2}+1, \pi+\frac{1}{2}\right\}$

26. a. None

b. None

c. $\left\{\frac{1}{2}+10.3\right\}$

d. $\left\{-\sqrt{2}, \pi+\sqrt{2}\right\}$

e. $\left\{-\sqrt{2}, \pi+\sqrt{2}, \frac{1}{2}+10.3\right\}$

27. a. 18.953 **b.** 18.952

28. a. 25.861 **b.** 25.861

29. a. 28.653 **b.** 28.653

30. a. 99.052 **b.** 99.052

31. a. 0.063 **b.** 0.062

32. a. 0.054 **b.** 0.053

33. a. 9.999 **b.** 9.998

34. a. 1.001 **b.** 1.000

35. a. 0.429 **b.** 0.428

36. a. 0.556 **b.** 0.555

37. a. 34.733 **b.** 34.733

38. a. 16.200 **b.** 16.200

39. $3+2=5$

40. $5 \cdot 2 = 10$

41. $x+2=3 \cdot 4$

42. $3+y=2+2$

43. $3y=1+2$

44. $2x=4 \cdot 6$

45. $x-2=6$

46. $2-y=6$

47. $\frac{x}{2}=6$

48. $\frac{2}{x}=6$

49. $9-4+2=5+2=7$

50. $6-4+3=2+3=5$

51. $-6+4 \cdot 3=-6+12=6$

52. $8-4 \cdot 2=8-8=0$

53. $4+5-8=9-8=1$

54. $8-3-4=5-4=1$

55. $4+\frac{1}{3}=\frac{12+1}{3}=\frac{13}{3}$

56. $2-\frac{1}{2}=\frac{4-1}{2}=\frac{3}{2}$

57.
$$\begin{aligned} 6-\left[3 \cdot 5+2 \cdot (3-2)\right] &= 6-\left[15+2 \cdot (1)\right] \\ &= 6-17 \\ &= -11 \end{aligned}$$

58. $2\cdot\left[8-3\cdot(4+2)\right]-3=2\cdot\left[8-3\cdot(6)\right]-3$
$=2\cdot[8-18]-3$
$=2\cdot[-10]-3$
$=-20-3$
$=-23$

59. $2\cdot(3-5)+8\cdot 2-1=2\cdot(-2)+16-1$
$=-4+16-1$
$=12-1$
$=11$

60. $1-(4\cdot 3-2+2)=1-(12-2+2)$
$=1-12$
$=-11$

61. $10-\left[6-2\cdot 2+(8-3)\right]\cdot 2=10-[6-4+5]\cdot 2$
$=10-[2+5]\cdot 2$
$=10-[7]\cdot 2$
$=10-14$
$=-4$

62. $2-5\cdot 4-\left[6\cdot(3-4)\right]=2-20-\left[6\cdot(-1)\right]$
$=-18-[-6]$
$=-18+6$
$=-12$

63. $(5-3)\frac{1}{2}=(2)\frac{1}{2}=1$

64. $(5+4)\frac{1}{3}=(9)\frac{1}{3}=3$

65. $\frac{4+8}{5-3}=\frac{12}{2}=6$

66. $\frac{2-4}{5-3}=\frac{-2}{2}=-1$

67. $\frac{3}{5}\cdot\frac{10}{21}=\frac{3\cdot 2\cdot 5}{5\cdot 3\cdot 7}=\frac{\cancel{3}\cdot 2\cdot\cancel{5}}{\cancel{5}\cdot\cancel{3}\cdot 7}=\frac{2}{7}$

68. $\frac{5}{9}\cdot\frac{3}{10}=\frac{5\cdot 3}{3\cdot 3\cdot 5\cdot 2}=\frac{\cancel{5}\cdot\cancel{3}}{\cancel{3}\cdot 3\cdot\cancel{5}\cdot 2}=\frac{1}{6}$

69. $\frac{6}{25}\cdot\frac{10}{27}=\frac{2\cdot 3\cdot 5\cdot 2}{5\cdot 5\cdot 3\cdot 9}=\frac{2\cdot\cancel{3}\cdot\cancel{5}\cdot 2}{\cancel{3}\cdot 5\cdot\cancel{3}\cdot 9}=\frac{4}{45}$

70. $\frac{21}{25}\cdot\frac{100}{3}=\frac{3\cdot 7\cdot 4\cdot 25}{25\cdot 3}=\frac{\cancel{3}\cdot 7\cdot 4\cdot\cancel{25}}{\cancel{25}\cdot\cancel{3}}=28$

71. $\frac{3}{4}+\frac{2}{5}=\frac{15+8}{20}=\frac{23}{20}$

72. $\frac{4}{3}+\frac{1}{2}=\frac{8+3}{6}=\frac{11}{6}$

73. $\frac{5}{6}+\frac{9}{5}=\frac{25+54}{30}=\frac{79}{30}$

74. $\frac{8}{9}+\frac{15}{2}=\frac{16+135}{18}=\frac{151}{18}$

75. $\frac{5}{18}+\frac{1}{12}=\frac{10+3}{36}=\frac{13}{36}$

76. $\frac{2}{15}+\frac{8}{9}=\frac{6+40}{45}=\frac{46}{45}$

77. $\frac{1}{30}-\frac{7}{18}=\frac{3-35}{90}=-\frac{32}{90}=-\frac{16}{45}$

78. $\frac{3}{14}-\frac{2}{21}=\frac{9-4}{42}=\frac{5}{42}$

79. $\frac{3}{20}-\frac{2}{15}=\frac{9-8}{60}=\frac{1}{60}$

80. $\frac{6}{35}-\frac{3}{14}=\frac{12-15}{70}=-\frac{3}{70}$

81. $\frac{\left(\frac{5}{18}\right)}{\left(\frac{11}{27}\right)}=\frac{5}{18}\cdot\frac{27}{11}=\frac{5\cdot 9\cdot 3}{9\cdot 2\cdot 11}=\frac{5\cdot\cancel{9}\cdot 3}{\cancel{9}\cdot 2\cdot 11}=\frac{15}{22}$

82. $\frac{\left(\frac{5}{21}\right)}{\left(\frac{2}{35}\right)}=\frac{5}{21}\cdot\frac{35}{2}=\frac{5\cdot 7\cdot 5}{7\cdot 3\cdot 2}=\frac{5\cdot\cancel{7}\cdot 5}{\cancel{7}\cdot 3\cdot 2}=\frac{25}{6}$

83. $\frac{1}{2}\cdot\frac{3}{5}+\frac{7}{10}=\frac{3}{10}+\frac{7}{10}=\frac{3+7}{10}=\frac{10}{10}=1$

84. $\frac{2}{3}+\frac{4}{5}\cdot\frac{1}{6}=\frac{2}{3}+\frac{2\cdot 2}{5\cdot 3\cdot 2}=\frac{2}{3}+\frac{2\cdot \cancel{2}}{5\cdot 3\cdot \cancel{2}}=\frac{2}{3}+\frac{2}{15}$
$=\frac{2}{3}\cdot\frac{5}{5}+\frac{2}{15}=\frac{10}{15}+\frac{2}{15}=\frac{10+2}{15}=\frac{12}{15}$
$=\frac{4\cdot 3}{5\cdot 3}=\frac{4\cdot \cancel{3}}{5\cdot \cancel{3}}=\frac{4}{5}$

85. $2\cdot\frac{3}{4}+\frac{3}{8}=\frac{2}{1}\cdot\frac{3}{4}+\frac{3}{8}=\frac{6}{4}+\frac{3}{8}=\frac{6}{4}\cdot\frac{2}{2}+\frac{3}{8}$
$=\frac{12}{8}+\frac{3}{8}=\frac{12+3}{8}=\frac{15}{8}$

86. $3\cdot\frac{5}{6}-\frac{1}{2}=\frac{3}{1}\cdot\frac{5}{6}-\frac{1}{2}=\frac{3\cdot 5}{3\cdot 2}-\frac{1}{2}=\frac{\cancel{3}\cdot 5}{\cancel{3}\cdot 2}-\frac{1}{2}$
$=\frac{5}{2}-\frac{1}{2}=\frac{5-1}{2}=\frac{4}{2}=2$

87. $6(x+4)=6x+24$

88. $4(2x-1)=8x-4$

89. $x(x-4)=x^2-4x$

90. $4x(x+3)=4x^2+12x$

91. $2\left(\frac{3}{4}x-\frac{1}{2}\right)=2\cdot\frac{3}{4}x-2\cdot\frac{1}{2}=\frac{2\cdot 3x}{2\cdot 2}-\frac{2}{2}$
$=\frac{\cancel{2}\cdot 3x}{\cancel{2}\cdot 2}-\frac{2}{2}=\frac{3}{2}x-1$

92. $3\left(\frac{2}{3}x+\frac{1}{6}\right)=3\cdot\frac{2}{3}x+3\cdot\frac{1}{6}=\frac{3\cdot 2x}{3}+\frac{3}{3\cdot 2}$
$=\frac{\cancel{3}\cdot 2x}{\cancel{3}}+\frac{\cancel{3}}{\cancel{3}\cdot 2}=2x+\frac{1}{2}$

93. $(x+2)(x+4)=x^2+4x+2x+8$
$=x^2+6x+8$

94. $(x+5)(x+1)=x^2+x+5x+5$
$=x^2+6x+5$

95. $(x-2)(x+1)=x^2+x-2x-2$
$=x^2-x-2$

96. $(x-4)(x+1)=x^2+x-4x-4$
$=x^2-3x-4$

97. $(x-8)(x-2)=x^2-2x-8x+16$
$=x^2-10x+16$

98. $(x-4)(x-2)=x^2-2x-4x+8$
$=x^2-6x+8$

99. $2x+3x=2\cdot x+3\cdot x$
$=(2+3)\cdot x$
$=(5)\cdot x$
$=5x$

100. $2+3\cdot 4=2+12=14$
since multiplication comes before addition in the order of operations for real numbers.

$(2+3)\cdot 4=5\cdot 4=20$
since operations inside parentheses come before multiplication in the order of operations for real numbers.

101. $2(3\cdot 4)=2(12)=24$
$(2\cdot 3)\cdot(2\cdot 4)=(6)(8)=48$

102. $\frac{4+3}{2+5}=\frac{7}{7}=1$, but
$\frac{4}{2}+\frac{3}{5}=\frac{4\cdot 5+3\cdot 2}{10}=\frac{20+6}{10}=\frac{26}{10}=\frac{13}{5}=2.6$

103. Subtraction is not commutative; for example: $2-3=-1\neq 1=3-2$.

104. Subtraction is not associative; for example: $(5-2)-1=2\neq 4=5-(2-1)$.

105. Division is not commutative; for example: $\frac{2}{3}\neq\frac{3}{2}$.

106. Division is not associative; for example: $(12\div 2)\div 2=6\div 2=3$, but
$12\div(2\div 2)=12\div 1=12$.

107. The Symmetric Property implies that if $2 = x$, then $x = 2$.

108. From the *principle of substitution,* if $x = 5$, then

$$(x)(x) = (5)(5)$$
$$\Rightarrow x^2 = 25$$
$$\Rightarrow x^2 + x = 25 + 5$$
$$\Rightarrow x^2 + x = 30$$

109. There are no real numbers that are both rational and irrational, since an irrational number, by definition, is a number that cannot be expressed as the ratio of two integers; that is, not a rational number

Every real number is either a rational number or an irrational number, since the decimal form of a real number either involves an infinitely repeating pattern of digits or an infinite, non-repeating string of digits.

110. The sum of an irrational number and a rational number must be irrational. Otherwise, the irrational number would then be the difference of two rational numbers, and therefore would have to be rational.

111. Answers will vary.

112. Since 1 day = 24 hours, we compute $\frac{12997}{24} = 541.541\overline{6}$.
Now we only need to consider the decimal part of the answer in terms of a 24 hour day. That is, $\left(0.541\overline{6}\right)(24) \approx 13$ hours. So it must be 13 hours later than 12 noon, which makes the time 1 AM CST.

113. Answers will vary.

Section R.2

1. variable

2. origin

3. strict

4. base; exponent (or power)

5. 1.2345678×10^3

6. True.

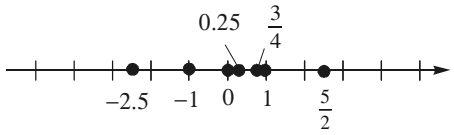
7. True

8. False; the absolute value of a real number is nonnegative. $|0| = 0$ which is not a positive number.

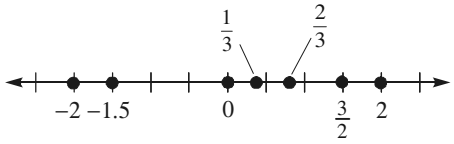
9. False; a number in scientific notation is expressed as the product of a number, x, $1 \le x < 10$ or $-10 < x \le -1$, and a power of 10.

10. False; to multiply two expressions with the same base, retain the base and *add* the exponents.

11.



12.



13. $\frac{1}{2} > 0$

14. $5 < 6$

15. $-1 > -2$

16. $-3 < -\frac{5}{2}$

17. $\pi > 3.14$

18. $\sqrt{2} > 1.41$

19. $\frac{1}{2} = 0.5$

20. $\frac{1}{3} > 0.33$

21. $\frac{2}{3} < 0.67$

22. $\frac{1}{4} = 0.25$

23. $x > 0$

24. $z < 0$

25. $x < 2$

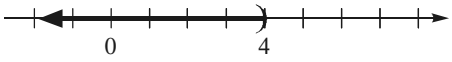
26. $y > -5$

27. $x \le 1$

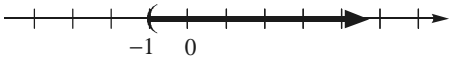
28. $x \ge 2$

29. Graph on the number line: $x \ge -2$

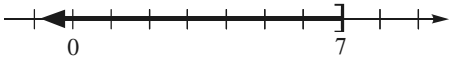
30. Graph on the number line: $x < 4$



31. Graph on the number line: $x > -1$



32. Graph on the number line: $x \le 7$



33. $d(C,D) = d(0,1) = |1-0| = |1| = 1$

34. $d(C,A) = d(0,-3) = |-3-0| = |-3| = 3$

35. $d(D,E) = d(1,3) = |3-1| = |2| = 2$

36. $d(C,E) = d(0,3) = |3-0| = |3| = 3$

37. $d(A,E) = d(-3,3) = |3-(-3)| = |6| = 6$

38. $d(D,B) = d(1,-1) = |-1-1| = |-2| = 2$

39. $x + 2y = -2 + 2 \cdot 3 = -2 + 6 = 4$

40. $3x + y = 3(-2) + 3 = -6 + 3 = -3$

41. $5xy + 2 = 5(-2)(3) + 2 = -30 + 2 = -28$

42. $-2x + xy = -2(-2) + (-2)(3) = 4 - 6 = -2$

43. $\dfrac{2x}{x-y} = \dfrac{2(-2)}{-2-3} = \dfrac{-4}{-5} = \dfrac{4}{5}$

44. $\dfrac{x+y}{x-y} = \dfrac{-2+3}{-2-3} = \dfrac{1}{-5} = -\dfrac{1}{5}$

45. $\dfrac{3x+2y}{2+y} = \dfrac{3(-2)+2(3)}{2+3} = \dfrac{-6+6}{5} = \dfrac{0}{5} = 0$

46. $\dfrac{2x-3}{y} = \dfrac{2(-2)-3}{3} = \dfrac{-4-3}{3} = -\dfrac{7}{3}$

47. $|x+y| = |3+(-2)| = |1| = 1$

48. $|x-y| = |3-(-2)| = |5| = 5$

49. $|x| + |y| = |3| + |-2| = 3 + 2 = 5$

50. $|x| - |y| = |3| - |-2| = 3 - 2 = 1$

51. $\dfrac{|x|}{x} = \dfrac{|3|}{3} = \dfrac{3}{3} = 1$

52. $\dfrac{|y|}{y} = \dfrac{|-2|}{-2} = \dfrac{2}{-2} = -1$

53.
$$\begin{aligned} |4x-5y| &= |4(3)-5(-2)| \\ &= |12+10| \\ &= |22| \\ &= 22 \end{aligned}$$

54. $|3x+2y| = |3(3)+2(-2)| = |9-4| = |5| = 5$

55.
$$\begin{aligned} ||4x|-|5y|| &= ||4(3)|-|5(-2)|| \\ &= ||12|-|-10|| \\ &= |12-10| \\ &= |2| \\ &= 2 \end{aligned}$$

56.
$$\begin{aligned} 3|x|+2|y| &= 3|3|+2|-2| \\ &= 3 \cdot 3 + 2 \cdot 2 \\ &= 9 + 4 \\ &= 13 \end{aligned}$$

57. $\dfrac{x^2-1}{x}$

Part (c) must be excluded. The value $x = 0$ must be excluded from the domain because it causes division by 0.

58. $\dfrac{x^2+1}{x}$

Part (c) must be excluded. The value $x=0$ must be excluded from the domain because it causes division by 0.

59. $\dfrac{x}{x^2-9}=\dfrac{x}{(x-3)(x+3)}$

Part (a) must be excluded. The values $x=-3$ and $x=3$ must be excluded from the domain because they cause division by 0.

60. $\dfrac{x}{x^2+9}$

None of the given values are excluded. The domain is all real numbers.

61. $\dfrac{x^2}{x^2+1}$

None of the given values are excluded. The domain is all real numbers.

62. $\dfrac{x^3}{x^2-1}=\dfrac{x^3}{(x-1)(x+1)}$

Parts (b) and (d) must be excluded. The values $x=1$, and $x=-1$ must be excluded from the domain because they cause division by 0.

63. $\dfrac{x^2+5x-10}{x^3-x}=\dfrac{x^2+5x-10}{x(x-1)(x+1)}$

Parts (b), (c), and (d) must be excluded. The values $x=0$, $x=1$, and $x=-1$ must be excluded from the domain because they cause division by 0.

64. $\dfrac{-9x^2-x+1}{x^3+x}=\dfrac{-9x^2-x+1}{x(x^2+1)}$

Part (c) must be excluded. The value $x=0$ must be excluded from the domain because it causes division by 0.

65. $\dfrac{4}{x-5}$

$x=5$ must be exluded because it makes the denominator equal 0.

Domain $=\{x\,|\,x\neq 5\}$

66. $\dfrac{-6}{x+4}$

$x=-4$ must be excluded sine it makes the denominator equal 0.

Domain $=\{x\,|\,x\neq -4\}$

67. $\dfrac{x}{x+4}$

$x=-4$ must be excluded sine it makes the denominator equal 0.

Domain $=\{x\,|\,x\neq -4\}$

68. $\dfrac{x-2}{x-6}$

$x=6$ must be excluded sine it makes the denominator equal 0.

Domain $=\{x\,|\,x\neq 6\}$

69. $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(32-32)=\dfrac{5}{9}(0)=0°\text{C}$

70. $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(212-32)=\dfrac{5}{9}(180)=100°\text{C}$

71. $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(77-32)=\dfrac{5}{9}(45)=25°\text{C}$

72. $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(-4-32)$

$=\dfrac{5}{9}(-36)$

$=-20°\text{C}$

73. $(-4)^2=(-4)(-4)=16$

74. $-4^2=-(4)^2=-16$

75. $4^{-2}=\dfrac{1}{4^2}=\dfrac{1}{16}$

76. $-4^{-2}=-\dfrac{1}{4^2}=-\dfrac{1}{16}$

77. $3^{-6}\cdot 3^4=3^{-6+4}=3^{-2}=\dfrac{1}{3^2}=\dfrac{1}{9}$

78. $4^{-2}\cdot 4^3=4^{-2+3}=4^1=4$

79. $\left(3^{-2}\right)^{-1}=3^{(-2)(-1)}=3^2=9$

80. $\left(2^{-1}\right)^{-3} = 2^{(-1)(-3)} = 2^3 = 8$

81. $\sqrt{25} = \sqrt{5^2} = 5$

82. $\sqrt{36} = \sqrt{6^2} = 6$

83. $\sqrt{(-4)^2} = |-4| = 4$

84. $\sqrt{(-3)^2} = |-3| = 3$

85. $\left(8x^3\right)^2 = 8^2\left(x^3\right)^2 = 64x^6$

86. $\left(-4x^2\right)^{-1} = \dfrac{1}{-4x^2} = -\dfrac{1}{4x^2}$

87. $\left(x^2y^{-1}\right)^2 = \left(x^2\right)^2 \cdot \left(y^{-1}\right)^2 = x^4y^{-2} = \dfrac{x^4}{y^2}$

88. $\left(x^{-1}y\right)^3 = \left(x^{-1}\right)^3 \cdot y^3 = x^{-3}y^3 = \dfrac{y^3}{x^3}$

89. $\dfrac{x^2y^3}{xy^4} = x^{2-1}y^{3-4} = x^1y^{-1} = \dfrac{x}{y}$

90. $\dfrac{x^{-2}y}{xy^2} = x^{-2-1}y^{1-2} = x^{-3}y^{-1} = \dfrac{1}{x^3y}$

91. $\dfrac{(-2)^3x^4(yz)^2}{3^2xy^3z} = \dfrac{-8x^4y^2z^2}{9xy^3z}$

$= \dfrac{-8}{9}x^{4-1}y^{2-3}z^{2-1}$

$= \dfrac{-8}{9}x^3y^{-1}z^1$

$= -\dfrac{8x^3z}{9y}$

92. $\dfrac{4x^{-2}(yz)^{-1}}{2^3x^4y} = \dfrac{4x^{-2}y^{-1}z^{-1}}{8x^4y}$

$= \dfrac{4}{8}x^{-2-4}y^{-1-1}z^{-1}$

$= \dfrac{1}{2}x^{-6}y^{-2}z^{-1}$

$= \dfrac{1}{2x^6y^2z}$

93. $\left(\dfrac{3x^{-1}}{4y^{-1}}\right)^{-2} = \left(\dfrac{3y}{4x}\right)^{-2} = \left(\dfrac{4x}{3y}\right)^2 = \dfrac{4^2x^2}{3^2y^2} = \dfrac{16x^2}{9y^2}$

94. $\left(\dfrac{5x^{-2}}{6y^{-2}}\right)^{-3} = \left(\dfrac{5y^2}{6x^2}\right)^{-3} = \left(\dfrac{6x^2}{5y^2}\right)^3$

$= \dfrac{6^3\left(x^2\right)^3}{5^3\left(y^2\right)^3} = \dfrac{216x^6}{125y^6}$

95. $2xy^{-1} = \dfrac{2x}{y} = \dfrac{2(2)}{(-1)} = -4$

96. $-3x^{-1}y = \dfrac{-3y}{x} = \dfrac{-3(-1)}{(2)} = \dfrac{3}{2}$

97. $x^2 + y^2 = (2)^2 + (-1)^2 = 4 + 1 = 5$

98. $x^2y^2 = (2)^2(-1)^2 = 4 \cdot 1 = 4$

99. $(xy)^2 = \left(2 \cdot (-1)\right)^2 = (-2)^2 = 4$

100. $(x+y)^2 = \left(2 + (-1)\right)^2 = (1)^2 = 1$

101. $\sqrt{x^2} = |x| = |2| = 2$

102. $\left(\sqrt{x}\right)^2 = x = 2$

103. $\sqrt{x^2 + y^2} = \sqrt{(2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$

104. $\sqrt{x^2} + \sqrt{y^2} = |x| + |y| = |2| + |-1| = 2 + 1 = 3$

105. $x^y = 2^{-1} = \dfrac{1}{2}$

106. $y^x = (-1)^2 = 1$

107. If $x = 2$,

$$\begin{aligned} 2x^3 - 3x^2 + 5x - 4 &= 2 \cdot 2^3 - 3 \cdot 2^2 + 5 \cdot 2 - 4 \\ &= 16 - 12 + 10 - 4 \\ &= 10 \end{aligned}$$

If $x = 1$,

$$\begin{aligned} 2x^3 - 3x^2 + 5x - 4 &= 2 \cdot 1^3 - 3 \cdot 1^2 + 5 \cdot 1 - 4 \\ &= 2 - 3 + 5 - 4 \\ &= 0 \end{aligned}$$

108. If $x = 1$,

$$\begin{aligned} 4x^3 + 3x^2 - x + 2 &= 4 \cdot 1^3 + 3 \cdot 1^2 - 1 + 2 \\ &= 4 + 3 - 1 + 2 \\ &= 8 \end{aligned}$$

If $x = 2$,

$$\begin{aligned} 4x^3 + 3x^2 - x + 2 &= 4 \cdot 2^3 + 3 \cdot 2^2 - 2 + 2 \\ &= 32 + 12 - 2 + 2 \\ &= 44 \end{aligned}$$

109. $\dfrac{(666)^4}{(222)^4} = \left(\dfrac{666}{222}\right)^4 = 3^4 = 81$

110. $$\begin{aligned} (0.1)^3 (20)^3 &= \left(\frac{1}{10}\right)^3 \cdot (2 \cdot 10)^3 \\ &= \frac{1}{10^3} \cdot 2^3 \cdot 10^3 \\ &= 2^3 = 8 \end{aligned}$$

111. $(8.2)^6 \approx 304,006.671$

112. $(3.7)^5 \approx 693.440$

113. $(6.1)^{-3} \approx 0.004$

114. $(2.2)^{-5} \approx 0.019$

115. $(-2.8)^6 \approx 481.890$

116. $-(2.8)^6 \approx -481.890$

117. $(-8.11)^{-4} \approx 0.000$

118. $-(8.11)^{-4} \approx -0.000$

119. $454.2 = 4.542 \times 10^2$

120. $32.14 = 3.214 \times 10^1$

121. $0.013 = 1.3 \times 10^{-2}$

122. $0.00421 = 4.21 \times 10^{-3}$

123. $32,155 = 3.2155 \times 10^4$

124. $21,210 = 2.121 \times 10^4$

125. $0.000423 = 4.23 \times 10^{-4}$

126. $0.0514 = 5.14 \times 10^{-2}$

127. $6.15 \times 10^4 = 61,500$

128. $9.7 \times 10^3 = 9700$

129. $1.214 \times 10^{-3} = 0.001214$

130. $9.88 \times 10^{-4} = 0.000988$

131. $1.1 \times 10^8 = 110,000,000$

132. $4.112 \times 10^2 = 411.2$

133. $8.1 \times 10^{-2} = 0.081$

134. $6.453 \times 10^{-1} = 0.6453$

135. $A = lw$

136. $P = 2(l + w)$

137. $C = \pi d$

138. $A = \dfrac{1}{2}bh$

139. $A = \dfrac{\sqrt{3}}{4}x^2$

140. $P = 3x$

141. $V = \dfrac{4}{3}\pi r^3$

142. $S = 4\pi r^2$

143. $V = x^3$

144. $S = 6x^2$

145. **a.** If $x = 1000$,

$$\begin{aligned} C &= 4000 + 2x \\ &= 4000 + 2(1000) \\ &= 4000 + 2000 \\ &= \$6000 \end{aligned}$$

The cost of producing 1000 watches is \$6000.

b. If $x = 2000$,

$$\begin{aligned} C &= 4000 + 2x \\ &= 4000 + 2(2000) \\ &= 4000 + 4000 \\ &= \$8000 \end{aligned}$$

The cost of producing 2000 watches is \$8000.

146. $210 + 80 - 120 + 25 - 60 - 32 - 5 = \$98$
His balance at the end of the month was \$98.

147. We want the difference between x and 4 to be at least 6 units. Since we don't care whether the value for x is larger or smaller than 4, we take the absolute value of the difference. We want the inequality to be non-strict since we are dealing with an 'at least' situation. Thus, we have
$|x - 4| \geq 6$

148. We want the difference between x and 2 to be more than 5 units. Since we don't care whether the value for x is larger or smaller than 2, we take the absolute value of the difference. We want the inequality to be strict since we are dealing with a 'more than' situation. Thus, we have
$|x - 2| > 5$

149. **a.** $|x - 110| = |108 - 110| = |-2| = 2 \leq 5$
108 volts is acceptable.

b. $|x - 110| = |104 - 110| = |-6| = 6 > 5$
104 volts is *not* acceptable.

150. **a.** $|x - 220| = |214 - 220| = |-6| = 6 \leq 8$
214 volts is acceptable.

b. $|x - 220| = |209 - 220| = |-11| = 11 > 8$
209 volts is *not* acceptable.

151. **a.**

$$\begin{aligned} |x - 3| &= |2.999 - 3| \\ &= |-0.001| \\ &= 0.001 \leq 0.01 \end{aligned}$$

A radius of 2.999 centimeters is acceptable.

b.

$$\begin{aligned} |x - 3| &= |2.89 - 3| \\ &= |-0.11| \\ &= 0.11 \not\leq 0.01 \end{aligned}$$

A radius of 2.89 centimeters is *not* acceptable.

152. **a.**

$$\begin{aligned} |x - 98.6| &= |97 - 98.6| \\ &= |-1.6| \\ &= 1.6 \geq 1.5 \end{aligned}$$

97°F is unhealthy.

b.

$$\begin{aligned} |x - 98.6| &= |100 - 98.6| \\ &= |1.4| \\ &= 1.4 < 1.5 \end{aligned}$$

100°F is *not* unhealthy.

153. The distance from Earth to the Moon is about $4 \times 10^8 = 400,000,000$ meters.

154. The height of Mt. Everest is about $8848 = 8.848 \times 10^3$ meters.

155. The wavelength of visible light is about $5 \times 10^{-7} = 0.0000005$ meters.

156. The diameter of an atom is about $1 \times 10^{-10} = 0.0000000001$ meters.

157. The smallest commercial copper wire has a diameter of about $0.0005 = 5 \times 10^{-4}$ inches.

158. The smallest motor ever made is less than $0.05 = 5 \times 10^{-2}$ centimeters wide.

159. $186{,}000 \cdot 60 \cdot 60 \cdot 24 \cdot 365$

$= \left(1.86 \times 10^5\right)\left(6 \times 10^1\right)^2 \left(2.4 \times 10^1\right)\left(3.65 \times 10^2\right)$

$= 586.5696 \times 10^{10} = 5.865696 \times 10^{12}$

There are about 5.9×10^{12} miles in one light-year.

160. $\dfrac{93{,}000{,}000}{186{,}000} = \dfrac{9.3 \times 10^7}{1.86 \times 10^5} = 5 \times 10^2$

$= 500$ seconds ≈ 8 min. 20 sec.

It takes about 8 minutes 20 seconds for a beam of light to reach Earth from the Sun.

161. $\dfrac{1}{3} = 0.333333 \ldots > 0.333$

$\dfrac{1}{3}$ is larger by approximately $0.0003333 \ldots$

162. $\dfrac{2}{3} = 0.666666 \ldots > 0.666$

$\dfrac{2}{3}$ is larger by approximately $0.0006666 \ldots$

163. No. For any positive number a, the value $\dfrac{a}{2}$ is smaller and therefore closer to 0.

164. We are given that $1 < x^2 < 10$. This implies that $1 < x < \sqrt{10}$. Since $x < \sqrt{10} \approx 3.162$ and $x > \pi \approx 3.142$, the number could be 3.15 or 3.16 (which are between 1 and 10 as required). The number could also be 3.14 since numbers such as 3.146 which lie between π and $\sqrt{10}$ would equal 3.14 when truncated to two decimal places.

165. Answers will vary.

166. Answers will vary.
$5 < 8$ is a true statement because 5 is further to the left than 8 on a real number line.

Section R.3

1. right; hypotenuse

2. $A = \dfrac{1}{2}bh$

3. $C = 2\pi r$

4. similar

5. True.

6. True. $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

7. False; the volume of a sphere of radius r is given by $V = \dfrac{4}{3}\pi r^3$.

8. True. The lengths of the corresponding sides are equal.

9. True. Two corresponding angles are equal.

10. False. The sides are not proportional.

11. $a = 5,\ b = 12,$

$c^2 = a^2 + b^2$

$= 5^2 + 12^2$

$= 25 + 144$

$= 169 \Rightarrow c = 13$

12. $a = 6,\ b = 8,$

$c^2 = a^2 + b^2$

$= 6^2 + 8^2$

$= 36 + 64$

$= 100 \Rightarrow c = 10$

13. $a = 10,\ b = 24,$

$c^2 = a^2 + b^2$

$= 10^2 + 24^2$

$= 100 + 576$

$= 676 \Rightarrow c = 26$

14. $a = 4,\ b = 3,$

$c^2 = a^2 + b^2$

$= 4^2 + 3^2$

$= 16 + 9$

$= 25 \Rightarrow c = 5$

15. $a = 7,\ b = 24,$

$c^2 = a^2 + b^2$

$= 7^2 + 24^2$

$= 49 + 576$

$= 625 \Rightarrow c = 25$

16. $a = 14,\ b = 48,$

$c^2 = a^2 + b^2$

$= 14^2 + 48^2$

$= 196 + 2304$

$= 2500 \Rightarrow c = 50$

17. $5^2 = 3^2 + 4^2$

$25 = 9 + 16$

$25 = 25$

The given triangle is a right triangle. The hypotenuse is 5.

18. $10^2 = 6^2 + 8^2$

$100 = 36 + 64$

$100 = 100$

The given triangle is a right triangle. The hypotenuse is 10.

19. $6^2 = 4^2 + 5^2$

$36 = 16 + 25$

$36 = 41$ false

The given triangle is not a right triangle.

20. $3^2 = 2^2 + 2^2$

$9 = 4 + 4$

$9 = 8$ false

The given triangle is not a right triangle.

21. $25^2 = 7^2 + 24^2$

$625 = 49 + 576$

$625 = 625$

The given triangle is a right triangle. The hypotenuse is 25.

22. $26^2 = 10^2 + 24^2$

$676 = 100 + 576$

$676 = 676$

The given triangle is a right triangle. The hypotenuse is 26.

23. $6^2 = 3^2 + 4^2$

$36 = 9 + 16$

$36 = 25$ false

The given triangle is not a right triangle.

24. $7^2 = 5^2 + 4^2$

$49 = 25 + 16$

$49 = 41$ false

The given triangle is not a right triangle.

25. $A = l \cdot w = 4 \cdot 2 = 8 \text{ in}^2$

26. $A = l \cdot w = 9 \cdot 4 = 36 \text{ cm}^2$

27. $A = \frac{1}{2} b \cdot h = \frac{1}{2}(2)(4) = 4 \text{ in}^2$

28. $A = \frac{1}{2} b \cdot h = \frac{1}{2}(4)(9) = 18 \text{ cm}^2$

29. $A = \pi r^2 = \pi(5)^2 = 25\pi \text{ m}^2$

$C = 2\pi r = 2\pi(5) = 10\pi \text{ m}$

30. $A = \pi r^2 = \pi(2)^2 = 4\pi \text{ ft}^2$

$C = 2\pi r = 2\pi(2) = 4\pi \text{ ft}$

31. $V = l\,w\,h = 8 \cdot 4 \cdot 7 = 224 \text{ ft}^3$

$S = 2lw + 2lh + 2wh$

$= 2(8)(4) + 2(8)(7) + 2(4)(7)$

$= 64 + 112 + 56$

$= 232 \text{ ft}^2$

32. $V = l\,w\,h = 9 \cdot 4 \cdot 8 = 288 \text{ in}^3$

$S = 2lw + 2lh + 2wh$

$= 2(9)(4) + 2(9)(8) + 2(4)(8)$

$= 72 + 144 + 64$

$= 280 \text{ in}^2$

33. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 4^3 = \frac{256}{3}\pi \text{ cm}^3$

$S = 4\pi r^2 = 4\pi \cdot 4^2 = 64\pi \text{ cm}^2$

34. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 3^3 = 36\pi \text{ ft}^3$

$S = 4\pi r^2 = 4\pi \cdot 3^2 = 36\pi \text{ ft}^2$

35. $V = \pi r^2 h = \pi(9)^2(8) = 648\pi \text{ in}^3$

$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(9)^2 + 2\pi(9)(8) \\ &= 162\pi + 144\pi \\ &= 306\pi \text{ in}^2 \end{aligned}$$

36. $V = \pi r^2 h = \pi(8)^2(9) = 576\pi \text{ in}^3$

$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(8)^2 + 2\pi(8)(9) \\ &= 128\pi + 144\pi \\ &= 272\pi \text{ in}^2 \end{aligned}$$

37. The diameter of the circle is 2, so its radius is 1.

$A = \pi r^2 = \pi(1)^2 = \pi$ square units

38. The diameter of the circle is 2, so its radius is 1.

$A = 2^2 - \pi(1)^2 = 4 - \pi$ square units

39. The diameter of the circle is the length of the diagonal of the square.

$$\begin{aligned} d^2 &= 2^2 + 2^2 \\ &= 4 + 4 \\ &= 8 \\ d &= \sqrt{8} = 2\sqrt{2} \\ r &= \frac{d}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2} \end{aligned}$$

The area of the circle is:

$A = \pi r^2 = \pi\left(\sqrt{2}\right)^2 = 2\pi$ square units

40. The diameter of the circle is the length of the diagonal of the square.

$$\begin{aligned} d^2 &= 2^2 + 2^2 \\ &= 4 + 4 \\ &= 8 \\ d &= \sqrt{8} = 2\sqrt{2} \\ r &= \frac{d}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2} \end{aligned}$$

The area is:

$A = \pi\left(\sqrt{2}\right)^2 - 2^2 = 2\pi - 4$ square units

41. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\begin{aligned} \frac{8}{4} &= \frac{x}{2} \\ \frac{8 \cdot 2}{4} &= x \\ 4 &= x \end{aligned}$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 90°$, $B = 60°$, and $C = 30°$.

42. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\begin{aligned} \frac{6}{12} &= \frac{x}{16} \\ \frac{6 \cdot 16}{12} &= x \\ 8 &= x \end{aligned}$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 30°$, $B = 75°$, and $C = 75°$.

43. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\begin{aligned} \frac{30}{20} &= \frac{x}{45} \\ \frac{30 \cdot 45}{20} &= x \\ \frac{135}{2} &= x \text{ or } x = 67.5 \end{aligned}$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 60°$, $B = 95°$, and $C = 25°$.

44. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\begin{aligned} \frac{8}{10} &= \frac{x}{50} \\ \frac{8 \cdot 50}{10} &= x \\ 40 &= x \end{aligned}$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 50°$, $B = 125°$, and $C = 5°$.

45. The total distance traveled is 4 times the circumference of the wheel.

$$\text{Total Distance} = 4C = 4(\pi d) = 4\pi \cdot 16$$
$$= 64\pi \approx 201.1 \text{ inches} \approx 16.8 \text{ feet}$$

46. The distance traveled in one revolution is the circumference of the disk 4π.
The number of revolutions =

$$\frac{\text{dist. traveled}}{\text{circumference}} = \frac{20}{4\pi} = \frac{5}{\pi} \approx 1.6 \text{ revolutions}$$

47. Area of the border = area of EFGH – area of ABCD $= 10^2 - 6^2 = 100 - 36 = 64 \text{ ft}^2$

48. FG = 4 feet; BG = 4 feet and BC = 10 feet, so CG= 6 feet. The area of the triangle CGF is:

$$A = \frac{1}{2} \cdot (4)(6) = 12 \text{ ft}^2$$

49. Area of the window = area of the rectangle + area of the semicircle.

$$A = (6)(4) + \frac{1}{2} \cdot \pi \cdot 2^2 = 24 + 2\pi \approx 30.28 \text{ ft}^2$$

Perimeter of the window = 2 heights + width + one-half the circumference.

$$P = 2(6) + 4 + \frac{1}{2} \cdot \pi(4) = 12 + 4 + 2\pi$$
$$= 16 + 2\pi \approx 22.28 \text{ feet}$$

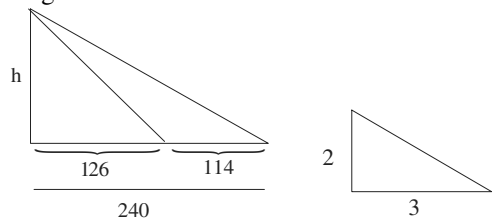
50. Area of the deck = area of the pool and deck – area of the pool.

$$A = \pi(13)^2 - \pi(10)^2 = 169\pi - 100\pi$$
$$= 69\pi \text{ ft}^2 \approx 216.77 \text{ ft}^2$$

The amount of fence is the circumference of the circle with radius 13 feet.

$$C = 2\pi(13) = 26\pi \text{ ft} \approx 81.68 \text{ ft}$$

51. We can form similar triangles using the Great Pyramid's height/shadow and Thales' height/shadow:



This allows us to write

$$\frac{h}{240} = \frac{2}{3}$$
$$h = \frac{2 \cdot 240}{3} = 160$$

The height of the Great Pyramid is 160 paces.

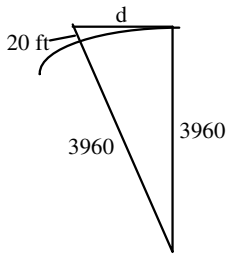
52. Let x = the approximate distance from San Juan to Hamilton and y = the approximate distance from Hamilton to Fort Lauderdale. Using similar triangles, we get

$$\frac{1046}{58} = \frac{x}{53.5} \qquad \frac{1046}{58} = \frac{y}{57}$$
$$\frac{1046 \cdot 53.5}{58} = x \qquad \frac{1046 \cdot 57}{58} = y$$
$$964.8 \approx x \qquad 1028.0 \approx y$$

The approximate distance between San Juan and Hamilton is 965 miles and the approximate distance between Hamilton and Fort Lauderdale is 1028 miles.

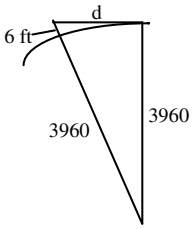
53. Convert 20 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$20 \text{ feet} = 20 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.003788 \text{ miles}$$
$$d^2 = (3960 + 0.003788)^2 - 3960^2 = 30 \text{ sq. miles}$$
$$d \approx 5.477 \text{ miles}$$



54. Convert 6 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$6 \text{ feet} = 6 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.001136 \text{ miles}$$
$$d^2 = (3960 + 0.001136)^2 - 3960^2 = 9 \text{ sq. miles}$$
$$d \approx 3 \text{ miles}$$



55. Convert 100 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$100 \text{ feet } = 100 \text{ feet } \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.018939 \text{ miles}$$

$$d^2 = (3960 + 0.018939)^2 - 3960^2 \approx 150 \text{ sq. miles}$$

$$d \approx 12.2 \text{ miles}$$

Convert 150 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$150 \text{ feet } = 150 \text{ feet } \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.028409 \text{ miles}$$

$$d^2 = (3960 + 0.028409)^2 - 3960^2 \approx 225 \text{ sq. miles}$$

$$d \approx 15.0 \text{ miles}$$

56. Given $m > 0$, $n > 0$ and $m > n$,

if $a = m^2 - n^2, b = 2mn$ and $c = m^2 + n^2$, then

$$a^2 + b^2 = \left(m^2 - n^2\right)^2 + (2mn)^2$$

$$= m^4 - 2m^2n^2 + n^4 + 4m^2n^2$$

$$= m^4 + 2m^2n^2 + n^4$$

and $c^2 = \left(m^2 + n^2\right)^2 = m^4 + 2m^2n^2 + n^4$

$\therefore a^2 + b^2 = c^2 \rightarrow a, b$ and c represent the sides of a right triangle.

57. Let l = length of the rectangle and w = width of the rectangle.

Notice that

$$(l + w)^2 - (l - w)^2$$

$$= [(l + w) + (l - w)][(l + w) - (l - w)]$$

$$= (2l)(2w) = 4lw = 4A$$

So $A = \frac{1}{4}[(l + w)^2 - (l - w)^2]$

Since $(l - w)^2 \geq 0$, the largest area will occur when $l - w = 0$ or $l = w$; that is, when the rectangle is a square. But

$$1000 = 2l + 2w = 2(l + w)$$

$$500 = l + w = 2l$$

$$250 = l = w$$

The largest possible area is $250^2 = 62500$ sq ft.

A circular pool with circumference = 1000 feet yields the equation: $2\pi r = 1000 \Rightarrow r = \dfrac{500}{\pi}$

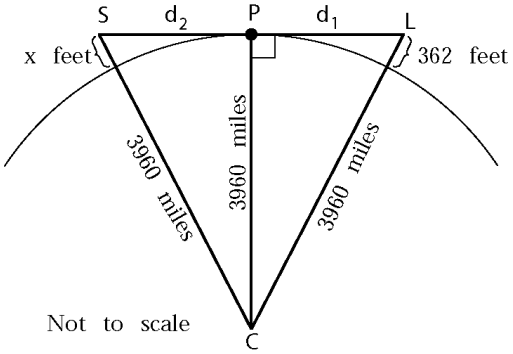
The area enclosed by the circular pool is:

$$A = \pi r^2 = \pi\left(\frac{500}{\pi}\right)^2 = \frac{500^2}{\pi} \approx 79577.47 \text{ ft}^2$$

Thus, a circular pool will enclose the most area.

58. Consider the diagram showing the lighthouse at point L, relative to the center of Earth, using the radius of Earth as 3960 miles. Let P refer to the furthest point on the horizon from which the light is visible. Note also that

$$362 \text{ feet} = \frac{362}{5280} \text{ miles.}$$



Apply the Pythagorean Theorem to ΔCPL:

$$(3960)^2 + (d_1)^2 = \left(3960 + \frac{362}{5280}\right)^2$$

$$(d_1)^2 = \left(3960 + \frac{362}{5280}\right)^2 - (3960)^2$$

$$d_1 = \sqrt{\left(3960 + \frac{362}{5280}\right)^2 - (3960)^2} \approx 23.30 \text{ mi.}$$

Therefore, the light from the lighthouse can be seen at point P on the horizon, where point P is approximately 23.30 miles away from the lighthouse. Brochure information is slightly overstated.

<u>Verify the ship information:</u>

Let S refer to the ship's location, and let x equal the height, in feet, of the ship.

We need $d_1 + d_2 \geq 40$.

Since $d_1 \approx 23.30$ miles we need

$d_2 \geq 40 - 23.30 = 16.70$ miles.

Apply the Pythagorean Theorem to ΔCPS :

$$(3960)^2 + (16.7)^2 = (3960 + x)^2$$

$$\sqrt{(3960)^2 + (16.7)^2} = 3960 + x$$

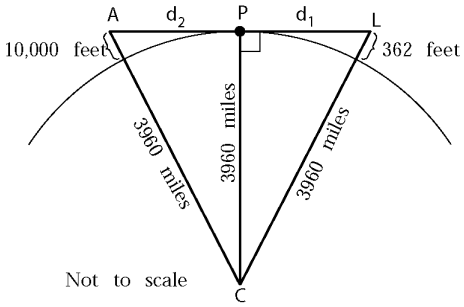
$$\sqrt{(3960)^2 + (16.7)^2} - 3960 = x$$

$$x \approx 0.035 \text{ miles}$$

$$x \approx 185.93 \text{ feet.}$$

The ship would have to be at least 186 feet tall to see the lighthouse from 40 miles away.

Verify the airplane information:



Let A refer to the airplane's location. The distance from the plane to point P is d_2.

We want to show that $d_1 + d_2 \geq 120$.

Assume the altitude of the airplane is

10,000 feet = $\frac{10000}{5280}$ miles.

Apply the Pythagorean Theorem to ΔCPA :

$$(3960)^2 + (d_2)^2 = \left(3960 + \frac{10000}{5280}\right)^2$$

$$(d_2)^2 = \left(3960 + \frac{10000}{5280}\right)^2 - (3960)^2$$

$$d_2 = \sqrt{\left(3960 + \frac{10000}{5280}\right)^2 - (3960)^2}$$

$$\approx 122.49 \text{ miles.}$$

Therefore,

$d_1 + d_2 \approx 23.30 + 122.49 = 145.79 \geq 120.$

The brochure information is slightly understated. Note that a plane at an altitude of 6233 feet could see the lighthouse from 120 miles away.

Section R.4

1. 4; 3

2. $x^4 - 16$

3. $x^3 - 8$

4. False; monomials cannot have negative degrees.

5. True

6. False; $x^3 + a^3 = (x + a)\left(x^2 - ax + a^2\right)$

7. $2x^3$ Monomial; Variable: x; Coefficient: 2; Degree: 3

8. $-4x^2$ Monomial; Variable: x; Coefficient: –4; Degree: 2

9. $\frac{8}{x} = 8x^{-1}$ Not a monomial; when written in the form ax^k, the variable has a negative exponent.

10. $-2x^{-3}$ Not a monomial; when written in the form ax^k, the variable has a negative exponent.

11. $-2xy^2$ Monomial; Variables: x, y; Coefficient: –2; Degree: 3

12. $5x^2y^3$ Monomial; Variables: x, y; Coefficient: 5; Degree: 5

13. $\frac{8x}{y} = 8xy^{-1}$ Not a monomial; when written in the form $ax^n y^m$, the exponent on the variable y is negative.

14. $-\frac{2x^2}{y^3} = -2x^2y^{-3}$ Not a monomial; when written in the form $ax^n y^m$, the exponent on the variable y is negative.

15. $x^2 + y^2$ Not a monomial; the expression contains more than one term. This expression is a binomial.

16. $3x^2+4$ Not a monomial; the expression contains more than one term. This expression is a binomial.

17. $3x^2-5$ Polynomial; Degree: 2

18. $1-4x$ Polynomial; Degree: 1

19. 5 Polynomial; Degree: 0

20. $-\pi$ Polynomial; Degree: 0

21. $3x^2-\frac{5}{x}$ Not a polynomial; the variable in the denominator results in an exponent that is not a nonnegative integer.

22. $\frac{3}{x}+2$ Not a polynomial; the variable in the denominator results in an exponent that is not a nonnegative integer.

23. $2y^3-\sqrt{2}$ Polynomial; Degree: 3

24. $10z^2+z$ Polynomial; Degree: 2

25. $\frac{x^2+5}{x^3-1}$ Not a polynomial; the polynomial in the denominator has a degree greater than 0.

26. $\frac{3x^3+2x-1}{x^2+x+1}$ Not a polynomial; the polynomial in the denominator has a degree greater than 0.

27. $(x^2+4x+5)+(3x-3)$
$=x^2+(4x+3x)+(5-3)$
$=x^2+7x+2$

28. $(x^3+3x^2+2)+(x^2-4x+4)$
$=x^3+(3x^2+x^2)+(-4x)+(2+4)$
$=x^3+4x^2-4x+6$

29. $(x^3-2x^2+5x+10)-(2x^2-4x+3)$
$=x^3-2x^2+5x+10-2x^2+4x-3$
$=x^3+(-2x^2-2x^2)+(5x+4x)+(10-3)$
$=x^3-4x^2+9x+7$

30. $(x^2-3x-4)-(x^3-3x^2+x+5)$
$=x^2-3x-4-x^3+3x^2-x-5$
$=-x^3+(x^2+3x^2)+(-3x-x)+(-4-5)$
$=-x^3+4x^2-4x-9$

31. $\left(6x^5+x^3+x\right)+\left(5x^4-x^3+3x^2\right)$
$=6x^5+5x^4+3x^2+x$

32. $\left(10x^5-8x^2\right)+\left(3x^3-2x^2+6\right)$
$=10x^5+3x^3-10x^2+6$

33. $(x^2-3x+1)+2(3x^2+x-4)$
$=x^2-3x+1+6x^2+2x-8$
$=7x^2-x-7$

34. $-2(x^2+x+1)+(-5x^2-x+2)$
$=-2x^2-2x-2-5x^2-x+2$
$=-7x^2-3x$

35. $6(x^3+x^2-3)-4(2x^3-3x^2)$
$=6x^3+6x^2-18-8x^3+12x^2$
$=-2x^3+18x^2-18$

36. $8(4x^3-3x^2-1)-6(4x^3+8x-2)$
$=32x^3-24x^2-8-24x^3-48x+12$
$=8x^3-24x^2-48x+4$

37. $\left(x^2-x+2\right)+\left(2x^2-3x+5\right)-\left(x^2+1\right)$
$=x^2-x+2+2x^2-3x+5-x^2-1$
$=2x^2-4x+6$

38. $\left(x^2+1\right)-\left(4x^2+5\right)+\left(x^2+x-2\right)$
$=x^2+1-4x^2-5+x^2+x-2$
$=-2x^2+x-6$

39. $9\left(y^2-3y+4\right)-6\left(1-y^2\right)$
$=9y^2-27y+36-6+6y^2$
$=15y^2-27y+30$

40. $8\left(1-y^3\right)+4\left(1+y+y^2+y^3\right)$
$=8-8y^3+4+4y+4y^2+4y^3$
$=-4y^3+4y^2+4y+12$

41. $x(x^2+x-4)=x^3+x^2-4x$

42. $4x^2(x^3-x+2)=4x^5-4x^3+8x^2$

43. $-2x^2(4x^3+5)=-8x^5-10x^2$

44. $5x^3(3x-4)=15x^4-20x^3$

45. $(x+1)(x^2+2x-4)$
$=x(x^2+2x-4)+1(x^2+2x-4)$
$=x^3+2x^2-4x+x^2+2x-4$
$=x^3+3x^2-2x-4$

46. $(2x-3)(x^2+x+1)$
$=2x(x^2+x+1)-3(x^2+x+1)$
$=2x^3+2x^2+2x-3x^2-3x-3$
$=2x^3-x^2-x-3$

47. $(x+2)(x+4)=x^2+4x+2x+8$
$=x^2+6x+8$

48. $(x+3)(x+5)=x^2+5x+3x+15$
$=x^2+8x+15$

49. $(2x+5)(x+2)=2x^2+4x+5x+10$
$=2x^2+9x+10$

50. $(3x+1)(2x+1)=6x^2+3x+2x+1$
$=6x^2+5x+1$

51. $(x-4)(x+2)=x^2+2x-4x-8$
$=x^2-2x-8$

52. $(x+4)(x-2)=x^2-2x+4x-8$
$=x^2+2x-8$

53. $(x-3)(x-2)=x^2-2x-3x+6$
$=x^2-5x+6$

54. $(x-5)(x-1)=x^2-x-5x+5$
$=x^2-6x+5$

55. $(2x+3)(x-2)=2x^2-4x+3x-6$
$=2x^2-x-6$

56. $(2x-4)(3x+1)=6x^2+2x-12x-4$
$=6x^2-10x-4$

57. $(-2x+3)(x-4)=-2x^2+8x+3x-12$
$=-2x^2+11x-12$

58. $(-3x-1)(x+1)=-3x^2-3x-x-1$
$=-3x^2-4x-1$

59. $(-x-2)(-2x-4)=2x^2+4x+4x+8$
$=2x^2+8x+8$

60. $(-2x-3)(3-x)=-6x+2x^2-9+3x$
$=2x^2-3x-9$

61. $(x-2y)(x+y)=x^2+xy-2xy-2y^2$
$=x^2-xy-2y^2$

62. $(2x+3y)(x-y)=2x^2-2xy+3xy-3y^2$
$=2x^2+xy-3y^2$

63. $(-2x-3y)(3x+2y)=-6x^2-4xy-9xy-6y^2$
$=-6x^2-13xy-6y^2$

64. $(x-3y)(-2x+y)=-2x^2+xy+6xy-3y^2$
$=-2x^2+7xy-3y^2$

65. $(x-7)(x+7)=x^2-7^2=x^2-49$

66. $(x-1)(x+1)=x^2-1^2=x^2-1$

67. $(2x+3)(2x-3)=(2x)^2-3^2=4x^2-9$

68. $(3x+2)(3x-2)=(3x)^2-2^2=9x^2-4$

69. $(x+4)^2=x^2+2\cdot x\cdot 4+4^2=x^2+8x+16$

70. $(x+5)^2=x^2+2\cdot x\cdot 5+5^2=x^2+10x+25$

71. $(x-4)^2=x^2-2\cdot x\cdot 4+4^2=x^2-8x+16$

72. $(x-5)^2=x^2-2\cdot x\cdot 5+5^2=x^2-10x+25$

73. $(3x+4)(3x-4)=(3x)^2-4^2=9x^2-16$

74. $(5x-3)(5x+3)=(5x)^2-3^2=25x^2-9$

75. $(2x-3)^2 = (2x)^2 - 2(2x)(3) + 3^2$
$= 4x^2 - 12x + 9$

76. $(3x-4)^2 = (3x)^2 - 2(3x)(4) + 4^2$
$= 9x^2 - 24x + 16$

77. $(x+y)(x-y) = (x)^2 - (y)^2 = x^2 - y^2$

78. $(x+3y)(x-3y) = (x)^2 - (3y)^2 = x^2 - 9y^2$

79. $(3x+y)(3x-y) = (3x)^2 - (y)^2 = 9x^2 - y^2$

80. $(3x+4y)(3x-4y) = (3x)^2 - (4y)^2 = 9x^2 - 16y^2$

81. $(x+y)^2 = x^2 + 2xy + y^2$

82. $(x-y)^2 = x^2 - 2xy + y^2$

83. $(x-2y)^2 = x^2 + 2(x\cdot(-2y)) + (2y)^2$
$= x^2 - 4xy + 4y^2$

84. $(2x+3y)^2 = (2x)^2 + 2(2x\cdot 3y) + (3y)^2$
$= 4x^2 + 12xy + 9y^2$

85. $(x-2)^3 = x^3 - 3\cdot x^2\cdot 2 + 3\cdot x\cdot 2^2 - 2^3$
$= x^3 - 6x^2 + 12x - 8$

86. $(x+1)^3 = x^3 + 3\cdot x^2\cdot 1 + 3\cdot x\cdot 1^2 + 1^3$
$= x^3 + 3x^2 + 3x + 1$

87. $(2x+1)^3 = (2x)^3 + 3(2x)^2(1) + 3(2x)\cdot 1^2 + 1^3$
$= 8x^3 + 12x^2 + 6x + 1$

88. $(3x-2)^3 = (3x)^3 - 3(3x)^2(2) + 3(3x)\cdot 2^2 - 2^3$
$= 27x^3 - 54x^2 + 36x - 8$

89.
$$\begin{array}{r} 4x^2 - 11x + 23 \\ x+2\overline{)\,4x^3 - 3x^2 + x + 1} \\ \underline{4x^3 + 8x^2} \\ -11x^2 + x \\ \underline{-11x^2 - 22x} \\ 23x + 1 \\ \underline{23x + 46} \\ -45 \end{array}$$

Check:
$(x+2)(4x^2 - 11x + 23) + (-45)$
$= 4x^3 - 11x^2 + 23x + 8x^2 - 22x + 46 - 45$
$= 4x^3 - 3x^2 + x + 1$
The quotient is $4x^2 - 11x + 23$; the remainder is –45.

90.
$$\begin{array}{r} 3x^2 - 7x + 15 \\ x+2\overline{)\,3x^3 - x^2 + x - 2} \\ \underline{3x^3 + 6x^2} \\ -7x^2 + x \\ \underline{-7x^2 - 14x} \\ 15x - 2 \\ \underline{15x + 30} \\ -32 \end{array}$$

Check:
$(x+2)(3x^2 - 7x + 15) + (-32)$
$= 3x^3 - 7x^2 + 15x + 6x^2 - 14x + 30 - 32$
$= 3x^3 - x^2 + x - 2$
The quotient is $3x^2 - 7x + 15$; the remainder is –32.

91.
$$\begin{array}{r} 4x - 3 \\ x^2\overline{)\,4x^3 - 3x^2 + x + 1} \\ \underline{4x^3} \\ -3x^2 + x + 1 \\ \underline{-3x^2} \\ x + 1 \end{array}$$

Check:
$(x^2)(4x-3) + (x+1) = 4x^3 - 3x^2 + x + 1$
The quotient is $4x - 3$; the remainder is $x + 1$.

92.
$$\begin{array}{r} 3x\ -\ 1 \\ x^2\overline{)\,3x^3-\ x^2+x-2} \\ \underline{3x^3} \\ -x^2+x-2 \\ \underline{-x^2} \\ x-\ 2 \end{array}$$

Check:

$(x^2)(3x-1)+(x-2)=3x^3-x^2+x-2$

The quotient is $3x-1$; the remainder is $x-2$.

93.
$$\begin{array}{r} 5x^2-13 \\ x^2+2\overline{)\,5x^4+0x^3-3x^2+x+1} \\ \underline{5x^4\quad\ +10x^2} \\ -13x^2+x+\ 1 \\ \underline{-13x^2\quad -26} \\ x+27 \end{array}$$

Check:

$\left(x^2+2\right)\left(5x^2-13\right)+(x+27)$

$=5x^4+10x^2-13x^2-26+x+27$

$=5x^4-3x^2+x+1$

The quotient is $5x^2-13$; the remainder is $x+27$.

94.
$$\begin{array}{r} 5x^2-11 \\ x^2+2\overline{)\,5x^4+0x^3-x^2+x-2} \\ \underline{5x^4\quad\ +10x^2} \\ -11x^2+x-\ 2 \\ \underline{-11x^2\quad -22} \\ x+20 \end{array}$$

Check:

$\left(x^2+2\right)\left(5x^2-11\right)+(x+20)$

$=5x^4+10x^2-11x^2-22+x+20$

$=5x^4-x^2+x-2$

The quotient is $5x^2-11$; the remainder is $x+20$.

95.
$$\begin{array}{r} 2x^2 \\ 2x^3-1\overline{)\,4x^5+0x^4+0x^3-3x^2+x+1} \\ \underline{4x^5\qquad\quad -2x^2} \\ -x^2+x+\ 1 \end{array}$$

Check:

$\left(2x^3-1\right)\left(2x^2\right)+\left(-x^2+x+1\right)$

$=4x^5-2x^2-x^2+x+1=4x^5-3x^2+x+1$

The quotient is $2x^2$; the remainder is $-x^2+x+1$.

96.
$$\begin{array}{r} x^2 \\ 3x^3-1\overline{)\,3x^5+0x^4+0x^3-x^2+x-2} \\ \underline{3x^5\qquad\quad -x^2} \\ x-2 \end{array}$$

Check:

$\left(3x^3-1\right)\left(x^2\right)+(x-2)=3x^5-x^2+x-2$

The quotient is x^2; the remainder is $x-2$.

97.
$$\begin{array}{r} x^2-2x+\frac{1}{2} \\ 2x^2+x+1\overline{)\,2x^4-3x^3+0x^2+x+1} \\ \underline{2x^4+\ x^3+\ x^2} \\ -4x^3-\ x^2+\ x \\ \underline{-4x^3-2x^2-2x} \\ x^2+3x+1 \\ \underline{x^2+\frac{1}{2}x+\frac{1}{2}} \\ \frac{5}{2}x+\frac{1}{2} \end{array}$$

Check:

$\left(2x^2+x+1\right)\left(x^2-2x+\frac{1}{2}\right)+\frac{5}{2}x+\frac{1}{2}$

$=2x^4-4x^3+x^2+x^3-2x^2+\frac{1}{2}x$

$+x^2-2x+\frac{1}{2}+\frac{5}{2}x+\frac{1}{2}$

$=2x^4-3x^3+x+1$

The quotient is $x^2-2x+\frac{1}{2}$; the remainder is $\frac{5}{2}x+\frac{1}{2}$.

98.
$$\begin{array}{r} x^2-\frac{2}{3}x-\frac{1}{9} \\ 3x^2+x+1\overline{)\,3x^4-x^3+0x^2+x-2} \\ \underline{3x^4+x^3+x^2} \\ -2x^3-x^2+x \\ \underline{-2x^3-\frac{2}{3}x^2-\frac{2}{3}x} \\ -\frac{1}{3}x^2+\frac{5}{3}x-2 \\ \underline{-\frac{1}{3}x^2-\frac{1}{9}x-\frac{1}{9}} \\ \frac{16}{9}x-\frac{17}{9} \end{array}$$

Check:

$$\left(3x^2+x+1\right)\left(x^2-\frac{2}{3}x-\frac{1}{9}\right)+\left(\frac{16}{9}x-\frac{17}{9}\right)$$
$$=3x^4+x^3+x^2-2x^3-\frac{2}{3}x^2-\frac{2}{3}x$$
$$-\frac{1}{3}x^2-\frac{1}{9}x-\frac{1}{9}+\frac{16}{9}x-\frac{17}{9}$$
$$=3x^4-x^3+x-2$$

The quotient is $x^2-\frac{2}{3}x-\frac{1}{9}$; the remainder is $\frac{16}{9}x-\frac{17}{9}$.

99.
$$\begin{array}{r} -4x^2-3x-3 \\ x-1\overline{)\,-4x^3+x^2+0x-4} \\ \underline{-4x^3+4x^2} \\ -3x^2 \\ \underline{-3x^2+3x} \\ -3x-4 \\ \underline{-3x+3} \\ -7 \end{array}$$

Check:

$$(x-1)(-4x^2-3x-3)+(-7)$$
$$=-4x^3-3x^2-3x+4x^2+3x+3-7$$
$$=-4x^3+x^2-4$$

The quotient is $-4x^2-3x-3$; the remainder is –7.

100.
$$\begin{array}{r} -3x^3-3x^2-3x-5 \\ x-1\overline{)\,-3x^4+0x^3+0x^2-2x-1} \\ \underline{-3x^4+3x^3} \\ -3x^3 \\ \underline{-3x^3+3x^2} \\ -3x^2-2x \\ \underline{-3x^2+3x} \\ -5x-1 \\ \underline{-5x+5} \\ -6 \end{array}$$

Check:

$$(x-1)(-3x^3-3x^2-3x-5)+(-6)$$
$$=-3x^4-3x^3-3x^2-5x+3x^3+3x^2$$
$$+3x+5-6$$
$$=-3x^4-2x-1$$

The quotient is $-3x^3-3x^2-3x-5$; the remainder is –6.

101.
$$\begin{array}{r} x^2-x-1 \\ x^2+x+1\overline{)\,x^4+0x^3-x^2+0x+1} \\ \underline{x^4+x^3+x^2} \\ -x^3-2x^2 \\ \underline{-x^3-x^2-x} \\ -x^2+x+1 \\ \underline{-x^2-x-1} \\ 2x+2 \end{array}$$

Check:

$$(x^2+x+1)(x^2-x-1)+2x+2$$
$$=x^4+x^3+x^2-x^3-x^2-x-x^2-x$$
$$-1+2x+2$$
$$=x^4-x^2+1$$

The quotient is x^2-x-1; the remainder is $2x+2$.

102.
$$\begin{array}{r} x^2+x-1 \\ x^2-x+1\overline{)\,x^4+0x^3-\ x^2+0x+1} \\ \underline{x^4-\ x^3+\ x^2} \\ x^3-2x^2 \\ \underline{x^3-\ x^2+x} \\ -x^2-x+1 \\ \underline{-x^2+x-1} \\ -2x+2 \end{array}$$

Check:

$$(x^2-x+1)(x^2+x-1)+(-2x+2)$$
$$=x^4+x^3-x^2-x^3-x^2+x+x^2+x$$
$$-1-2x+2$$
$$=x^4-x^2+1$$

The quotient is x^2+x-1; the remainder is $-2x+2$.

103.
$$\begin{array}{r} x^2+ax+a^2 \\ x-a\overline{)\,x^3+0x^2+\ 0x-a^3} \\ \underline{x^3-ax^2} \\ ax^2 \\ \underline{ax^2-a^2x} \\ a^2x-a^3 \\ \underline{a^2x-a^3} \\ 0 \end{array}$$

Check:

$$(x-a)(x^2+ax+a^2)+0$$
$$=x^3+ax^2+a^2x-ax^2-a^2x-a^3$$
$$=x^3-a^3$$

The quotient is x^2+ax+a^2; the remainder is 0.

104.
$$\begin{array}{r} x^4+ax^3+a^2x^2+a^3x+a^4 \\ x-a\overline{)\,x^5+0x^4+0x^3+\ 0x^2+\ 0x-a^5} \\ \underline{x^5-ax^4} \\ ax^4 \\ \underline{ax^4-a^2x^3} \\ a^2x^3 \\ \underline{a^2x^3-a^3x^2} \\ a^3x^2 \\ \underline{a^3x^2-a^4x} \\ a^4x-a^5 \\ \underline{a^4x-a^5} \\ 0 \end{array}$$

Check:

$$(x-a)(x^4+ax^3+a^2x^2+a^3x+a^4)+0$$
$$=x^5+ax^4+a^2x^3+a^3x^2+a^4x-ax^4$$
$$-a^2x^3-a^3x^2-a^4x-a^5$$
$$=x^5-a^5$$

The quotient is $x^4+ax^3+a^2x^2+a^3x+a^4$; the remainder is 0.

105. When we multiply polynomials $p_1(x)$ and $p_2(x)$, each term of $p_1(x)$ will be multiplied by each term of $p_2(x)$. So when the highest-powered term of $p_1(x)$ multiplies by the highest powered term of $p_2(x)$, the exponents on the variables in those terms will add according to the basic rules of exponents. Therefore, the highest powered term of the product polynomial will have degree equal to the sum of the degrees of $p_1(x)$ and $p_2(x)$.

106. When we add two polynomials $p_1(x)$ and $p_2(x)$, where the degree of $p_1(x) \neq$ the degree of $p_2(x)$, each term of $p_1(x)$ will be added to each term of $p_2(x)$. Since only the terms with equal degrees will combine via addition, the degree of the sum polynomial will be the degree of the highest powered term overall, that is, the degree of the polynomial that had the higher degree.

107. When we add two polynomials $p_1(x)$ and $p_2(x)$, where the degree of $p_1(x)$ = the degree of $p_2(x)$, the new polynomial will have degree $\le$ the degree of $p_1(x)$ and $p_2(x)$.

108. Answers will vary.

109. Answers will vary.

Section R.5

1. $3x(x-2)(x+2)$

2. prime

3. True; x^2+4 is prime over the set of real numbers.

4. False; $3x^3-2x^2-6x+4=(3x-2)(x^2-2)$

5. $3x+6=3(x+2)$

6. $7x-14=7(x-2)$

7. $ax^2+a=a(x^2+1)$

8. $ax-a=a(x-1)$

9. $x^3+x^2+x=x(x^2+x+1)$

10. $x^3-x^2+x=x(x^2-x+1)$

11. $2x^2-2x=2x(x-1)$

12. $3x^2-3x=3x(x-1)$

13. $3x^2y-6xy^2+12xy=3xy(x-2y+4)$

14. $60x^2y-48xy^2+72x^3y=12xy(5x-4y+6x^2)$

15. $x^2-1=x^2-1^2=(x-1)(x+1)$

16. $x^2-4=x^2-2^2=(x-2)(x+2)$

17. $4x^2-1=(2x)^2-1^2=(2x-1)(2x+1)$

18. $9x^2-1=(3x)^2-1^2=(3x-1)(3x+1)$

19. $x^2-16=x^2-4^2=(x-4)(x+4)$

20. $x^2-25=x^2-5^2=(x-5)(x+5)$

21. $25x^2-4=(5x-2)(5x+2)$

22. $36x^2-9=9(4x^2-1)=9(2x-1)(2x+1)$

23. $x^2+2x+1=(x+1)^2$

24. $x^2-4x+4=(x-2)^2$

25. $x^2+4x+4=(x+2)^2$

26. $x^2-2x+1=(x-1)^2$

27. $x^2-10x+25=(x-5)^2$

28. $x^2+10x+25=(x+5)^2$

29. $4x^2+4x+1=(2x+1)^2$

30. $9x^2+6x+1=(3x+1)^2$

31. $16x^2+8x+1=(4x+1)^2$

32. $25x^2+10x+1=(5x+1)^2$

33. $x^3-27=x^3-3^3=(x-3)(x^2+3x+9)$

34. $x^3+125=x^3+5^3=(x+5)(x^2-5x+25)$

35. $x^3+27=x^3+3^3=(x+3)(x^2-3x+9)$

36. $27-8x^3=3^3-(2x)^3$
$=(3-2x)(9+6x+4x^2)$
$=-(2x-3)(4x^2+6x+9)$

37. $8x^3+27=(2x)^3+3^3$
$=(2x+3)(4x^2-6x+9)$

38. $64-27x^3 = 4^3-(3x)^3$
$= (4-3x)(16+12x+9x^2)$
$= -(3x-4)(9x^2+12x+16)$

39. $x^2+5x+6 = (x+2)(x+3)$

40. $x^2+6x+8 = (x+2)(x+4)$

41. $x^2+7x+6 = (x+6)(x+1)$

42. $x^2+9x+8 = (x+8)(x+1)$

43. $x^2+7x+10 = (x+2)(x+5)$

44. $x^2+11x+10 = (x+10)(x+1)$

45. $x^2-10x+16 = (x-2)(x-8)$

46. $x^2-17x+16 = (x-16)(x-1)$

47. $x^2-7x-8 = (x+1)(x-8)$

48. $x^2-2x-8 = (x+2)(x-4)$

49. $x^2+7x-8 = (x+8)(x-1)$

50. $x^2+2x-8 = (x+4)(x-2)$

51. $2x^2+4x+3x+6 = 2x(x+2)+3(x+2)$
$= (x+2)(2x+3)$

52. $3x^2-3x+2x-2 = 3x(x-1)+2(x-1)$
$= (x-1)(3x+2)$

53. $2x^2-4x+x-2 = 2x(x-2)+1(x-2)$
$= (x-2)(2x+1)$

54. $3x^2+6x-x-2 = 3x(x+2)-1(x+2)$
$= (x+2)(3x-1)$

55. $6x^2+9x+4x+6 = 3x(2x+3)+2(2x+3)$
$= (2x+3)(3x+2)$

56. $9x^2-6x+3x-2 = 3x(3x-2)+1(3x-2)$
$= (3x-2)(3x+1)$

57. $3x^2+4x+1 = (3x+1)(x+1)$

58. $2x^2+3x+1 = (2x+1)(x+1)$

59. $2z^2+5z+3 = (2z+3)(z+1)$

60. $6z^2+5z+1 = (3z+1)(2z+1)$

61. $3x^2+2x-8 = (3x-4)(x+2)$

62. $3x^2+10x+8 = (3x+4)(x+2)$

63. $3x^2-2x-8 = (3x+4)(x-2)$

64. $3x^2-10x+8 = (3x-4)(x-2)$

65. $3x^2+14x+8 = (3x+2)(x+4)$

66. $3x^2-14x+8 = (3x-2)(x-4)$

67. $3x^2+10x-8 = (3x-2)(x+4)$

68. $3x^2-10x-8 = (3x+2)(x-4)$

69. Since B is 10 then we need half of 10 squared to be the last term in our trinomial. Thus
$\frac{1}{2}(10) = 5;\ (5)^2 = 25$
$x^2+10x+25 = (x+5)^2$

70. Since B is 14 then we need half of 14 squared to be the last term in our trinomial. Thus
$\frac{1}{2}(14) = 7;\ (7)^2 = 49$
$p^2+14p+49 = (p+7)^2$

71. Since B is -6 then we need half of -6 squared to be the last term in our trinomial. Thus
$\frac{1}{2}(-6) = -3;\ (-3)^2 = 9$
$y^2-6y+9 = (y-3)^2$

72. Since B is -4 then we need half of -4 squared to be the last term in our trinomial. Thus
$\frac{1}{2}(-4) = -2;\ (-2)^2 = 4$
$x^2 - 4x + 4 = (x-2)^2$

73. Since B is $-\frac{1}{2}$ then we need half of $-\frac{1}{2}$ squared to be the last term in our trinomial. Thus
$\frac{1}{2}(-\frac{1}{2}) = -\frac{1}{4};\ (-\frac{1}{4})^2 = \frac{1}{16}$
$x^2 - \frac{1}{2}x + \frac{1}{16} = (x-\frac{1}{4})^2$

74. Since B is $\frac{1}{3}$ then we need half of $\frac{1}{3}$ squared to be the last term in our trinomial. Thus
$\frac{1}{2}(\frac{1}{3}) = \frac{1}{6};\ (\frac{1}{6})^2 = \frac{1}{36}$
$x^2 + \frac{1}{3}x + \frac{1}{36} = (x+\frac{1}{6})^2$

75. $x^2 - 36 = (x-6)(x+6)$

76. $x^2 - 9 = (x-3)(x+3)$

77. $2 - 8x^2 = 2(1-4x^2) = 2\left(1-2x\right)\left(1+2x\right)$

78. $3 - 27x^2 = 3(1-9x^2) = 3\left(1-3x\right)\left(1+3x\right)$

79. $x^2 + 11x + 10 = (x+1)(x+10)$

80. $x^2 + 5x + 4 = (x+4)(x+1)$

81. $x^2 - 10x + 21 = \left(x-7\right)\left(x-3\right)$

82. $x^2 - 6x + 8 = (x-2)(x-4)$

83. $4x^2 - 8x + 32 = 4\left(x^2 - 2x + 8\right)$

84. $3x^2 - 12x + 15 = 3\left(x^2 - 4x + 5\right)$

85. $x^2 + 4x + 16$ is prime over the reals because there are no factors of 16 whose sum is 4.

86. $x^2 + 12x + 36 = (x+6)^2$

87. $15 + 2x - x^2 = -(x^2 - 2x - 15) = -(x-5)(x+3)$

88. $14 + 6x - x^2 = -(x^2 - 6x - 14)$ is prime over the integers because there are no factors of –14 whose sum is –6.

89. $3x^2 - 12x - 36 = 3(x^2 - 4x - 12)$
$= 3(x-6)(x+2)$

90. $x^3 + 8x^2 - 20x = x(x^2 + 8x - 20)$
$= x(x+10)(x-2)$

91. $y^4 + 11y^3 + 30y^2 = y^2(y^2 + 11y + 30)$
$= y^2(y+5)(y+6)$

92. $3y^3 - 18y^2 - 48y = 3y(y^2 - 6y - 16)$
$= 3y(y+2)(y-8)$

93. $4x^2 + 12x + 9 = (2x+3)^2$

94. $9x^2 - 12x + 4 = (3x-2)^2$

95. $6x^2 + 8x + 2 = 2\left(3x^2 + 4x + 1\right)$
$= 2\left(3x+1\right)\left(x+1\right)$

96. $8x^2 + 6x - 2 = 2\left(4x^2 + 3x - 1\right)$
$= 2\left(4x-1\right)\left(x+1\right)$

97. $x^4 - 81 = \left(x^2\right)^2 - 9^2 = (x^2-9)(x^2+9)$
$= (x-3)(x+3)(x^2+9)$

98. $x^4 - 1 = \left(x^2\right)^2 - 1^2 = (x^2-1)(x^2+1)$
$= (x-1)(x+1)(x^2+1)$

99. $x^6 - 2x^3 + 1 = (x^3-1)^2$
$= \left[(x-1)(x^2+x+1)\right]^2$
$= (x-1)^2(x^2+x+1)^2$

100. $x^6 + 2x^3 + 1 = (x^3+1)^2$
$= \left[(x+1)(x^2-x+1)\right]^2$
$= (x+1)^2(x^2-x+1)^2$

101. $x^7 - x^5 = x^5(x^2-1) = x^5(x-1)(x+1)$

102. $x^8 - x^5 = x^5(x^3 - 1) = x^5(x-1)(x^2 + x + 1)$

103. $16x^2 + 24x + 9 = (4x+3)^2$

104. $9x^2 - 24x + 16 = (3x-4)^2$

105. $5 + 16x - 16x^2 = -(16x^2 - 16x - 5)$
$= -(4x-5)(4x+1)$

106. $5 + 11x - 16x^2 = -(16x^2 - 11x - 5)$
$= -(16x+5)(x-1)$

107. $4y^2 - 16y + 15 = (2y-5)(2y-3)$

108. $9y^2 + 9y - 4 = (3y+4)(3y-1)$

109. $1 - 8x^2 - 9x^4 = -(9x^4 + 8x^2 - 1)$
$= -(9x^2 - 1)(x^2 + 1)$
$= -(3x-1)(3x+1)(x^2+1)$

110. $4 - 14x^2 - 8x^4 = -2(4x^4 + 7x^2 - 2)$
$= -2(4x^2 - 1)(x^2 + 2)$
$= -2(2x-1)(2x+1)(x^2+2)$

111. $x(x+3) - 6(x+3) = (x+3)(x-6)$

112. $5(3x-7) + x(3x-7) = (3x-7)(x+5)$

113. $(x+2)^2 - 5(x+2) = (x+2)\big[(x+2) - 5\big]$
$= (x+2)(x-3)$

114. $(x-1)^2 - 2(x-1) = (x-1)\big[(x-1) - 2\big]$
$= (x-1)(x-3)$

115. $(3x-2)^3 - 27$
$= (3x-2)^3 - 3^3$
$= \big[(3x-2) - 3\big]\big[(3x-2)^2 + 3(3x-2) + 9\big]$
$= (3x-5)\big(9x^2 - 12x + 4 + 9x - 6 + 9\big)$
$= (3x-5)\big(9x^2 - 3x + 7\big)$

116. $(5x+1)^3 - 1$
$= (5x+1)^3 - 1^3$
$= \big[(5x+1) - 1\big]\big[(5x+1)^2 + (1)(5x+1) + 1\big]$
$= 5x\big(25x^2 + 10x + 1 + 5x + 1 + 1\big)$
$= 5x\big(25x^2 + 15x + 3\big)$

117. $3\big(x^2 + 10x + 25\big) - 4(x+5)$
$= 3(x+5)^2 - 4(x+5)$
$= (x+5)\big[3(x+5) - 4\big]$
$= (x+5)(3x + 15 - 4)$
$= (x+5)(3x+11)$

118. $7\big(x^2 - 6x + 9\big) + 5(x-3)$
$= 7(x-3)^2 + 5(x-3)$
$= (x-3)\big[7(x-3) + 5\big]$
$= (x-3)(7x - 21 + 5)$
$= (x-3)(7x-16)$

119. $x^3 + 2x^2 - x - 2 = x^2(x+2) - 1(x+2)$
$= (x+2)(x^2 - 1)$
$= (x+2)(x-1)(x+1)$

120. $x^3 - 3x^2 - x + 3 = x^2(x-3) - 1(x-3)$
$= (x-3)(x^2 - 1)$
$= (x-3)(x-1)(x+1)$

121. $x^4 - x^3 + x - 1 = x^3(x-1) + 1(x-1)$
$= (x-1)(x^3 + 1)$
$= (x-1)(x+1)(x^2 - x + 1)$

122. $x^4 + x^3 + x + 1 = x^3(x+1) + 1(x+1)$
$= (x+1)(x^3 + 1)$
$= (x+1)(x+1)(x^2 - x + 1)$
$= (x+1)^2(x^2 - x + 1)$

123. $2(3x+4)^2+(2x+3)\cdot 2(3x+4)\cdot 3$
$=2(3x+4)\big((3x+4)+(2x+3)\cdot 3\big)$
$=2(3x+4)(3x+4+6x+9)$
$=2(3x+4)(9x+13)$

124. $5(2x+1)^2+(5x-6)\cdot 2(2x+1)\cdot 2$
$=(2x+1)\big(5(2x+1)+(5x-6)\cdot 4\big)$
$=(2x+1)(10x+5+20x-24)$
$=(2x+1)(30x-19)$

125. $2x(2x+5)+x^2\cdot 2=2x\big((2x+5)+x\big)$
$=2x(2x+5+x)$
$=2x(3x+5)$

126. $3x^2(8x-3)+x^3\cdot 8=x^2\big(3(8x-3)+8x\big)$
$=x^2(24x-9+8x)$
$=x^2(32x-9)$

127. $2(x+3)(x-2)^3+(x+3)^2\cdot 3(x-2)^2$
$=(x+3)(x-2)^2\big(2(x-2)+(x+3)\cdot 3\big)$
$=(x+3)(x-2)^2(2x-4+3x+9)$
$=(x+3)(x-2)^2(5x+5)$
$=5(x+3)(x-2)^2(x+1)$

128. $4(x+5)^3(x-1)^2+(x+5)^4\cdot 2(x-1)$
$=2(x+5)^3(x-1)\big(2(x-1)+(x+5)\big)$
$=2(x+5)^3(x-1)(2x-2+x+5)$
$=2(x+5)^3(x-1)(3x+3)$
$=2\cdot 3(x+5)^3(x-1)(x+1)$
$=6(x+5)^3(x-1)(x+1)$

129. $(4x-3)^2+x\cdot 2(4x-3)\cdot 4$
$=(4x-3)\big((4x-3)+8x\big)$
$=(4x-3)(4x-3+8x)$
$=(4x-3)(12x-3)$
$=3(4x-3)(4x-1)$

130. $3x^2(3x+4)^2+x^3\cdot 2(3x+4)\cdot 3$
$=3x^2(3x+4)\big((3x+4)+2x\big)$
$=3x^2(3x+4)(3x+4+2x)$
$=3x^2(3x+4)(5x+4)$

131. $2(3x-5)\cdot 3(2x+1)^3+(3x-5)^2\cdot 3(2x+1)^2\cdot 2$
$=6(3x-5)(2x+1)^2\big((2x+1)+(3x-5)\big)$
$=6(3x-5)(2x+1)^2(2x+1+3x-5)$
$=6(3x-5)(2x+1)^2(5x-4)$

132. $3(4x+5)^2\cdot 4(5x+1)^2+(4x+5)^3\cdot 2(5x+1)\cdot 5$
$=2(4x+5)^2(5x+1)\big(6(5x+1)+5(4x+5)\big)$
$=2(4x+5)^2(5x+1)(30x+6+20x+25)$
$=2(4x+5)^2(5x+1)(50x+31)$

133. Factors of 4: 1, 4 2, 2 –1, –4 –2, –2
Sum: 5 4 –5 –4
None of the sums of the factors is 0, so x^2+4 is prime.
Alternatively, the possibilities are $(x\pm 1)(x\pm 4)=x^2\pm 5x+4$ or $(x\pm 2)(x\pm 2)=x^2\pm 4x+4$, none of which equals x^2+4.

134. Factors of 1: 1, 1 –1, –1
Sum: 2 –2
None of the sums of the factors is 1, so x^2+x+1 is prime.
Alternatively, the possibilities are $(x\pm 1)^2=x^2\pm 2x+1$, neither of which equals x^2+x+1.

135. Answers will vary.

136. Answers will vary.

Section R.6

1. quotient; divisor; remainder

2. $-3\overline{)\,2 \quad 0 \quad -5 \quad 1}$

3. True

4. True

5.
$$\begin{array}{r|rrrr} 2 & 1 & -1 & 2 & 4 \\ & & 2 & 2 & 8 \\ \hline & 1 & 1 & 4 & 12 \end{array}$$
Quotient: $x^2 + x + 4$
Remainder: 12

6.
$$\begin{array}{r|rrrr} -1 & 1 & 2 & -3 & 1 \\ & & -1 & -1 & 4 \\ \hline & 1 & 1 & -4 & 5 \end{array}$$
Quotient: $x^2 + x - 4$
Remainder: 5

7.
$$\begin{array}{r|rrrr} 3 & 3 & 2 & -1 & 3 \\ & & 9 & 33 & 96 \\ \hline & 3 & 11 & 32 & 99 \end{array}$$
Quotient: $3x^2 + 11x + 32$
Remainder: 99

8.
$$\begin{array}{r|rrrr} -2 & -4 & 2 & -1 & 1 \\ & & 8 & -20 & 42 \\ \hline & -4 & 10 & -21 & 43 \end{array}$$
Quotient: $-4x^2 + 10x - 21$
Remainder: 43

9.
$$\begin{array}{r|rrrrrr} -3 & 1 & 0 & -4 & 0 & 1 & 0 \\ & & -3 & 9 & -15 & 45 & -138 \\ \hline & 1 & -3 & 5 & -15 & 46 & -138 \end{array}$$
Quotient: $x^4 - 3x^3 + 5x^2 - 15x + 46$
Remainder: -138

10.
$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 1 & 0 & 2 \\ & & 2 & 4 & 10 & 20 \\ \hline & 1 & 2 & 5 & 10 & 22 \end{array}$$
Quotient: $x^3 + 2x^2 + 5x + 10$
Remainder: 22

11.
$$\begin{array}{r|rrrrrrr} 1 & 4 & 0 & -3 & 0 & 1 & 0 & 5 \\ & & 4 & 4 & 1 & 1 & 2 & 2 \\ \hline & 4 & 4 & 1 & 1 & 2 & 2 & 7 \end{array}$$
Quotient: $4x^5 + 4x^4 + x^3 + x^2 + 2x + 2$
Remainder: 7

12.
$$\begin{array}{r|rrrrrr} -1 & 1 & 0 & 5 & 0 & 0 & -10 \\ & & -1 & 1 & -6 & 6 & -6 \\ \hline & 1 & -1 & 6 & -6 & 6 & -16 \end{array}$$
Quotient: $x^4 - x^3 + 6x^2 - 6x + 6$
Remainder: –16

13.
$$\begin{array}{r|rrrr} -1.1 & 0.1 & 0 & 0.2 & 0 \\ & & -0.11 & 0.121 & -0.3531 \\ \hline & 0.1 & -0.11 & 0.321 & -0.3531 \end{array}$$
Quotient: $0.1x^2 - 0.11x + 0.321$
Remainder: –0.3531

14.
$$\begin{array}{r|rrr} -2.1 & 0.1 & 0 & -0.2 \\ & & -0.21 & 0.441 \\ \hline & 0.1 & -0.21 & 0.241 \end{array}$$
Quotient: $0.1x - 0.21$
Remainder: 0.241

15.
$$\begin{array}{r|rrrrrr} 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 0 \end{array}$$
Quotient: $x^4 + x^3 + x^2 + x + 1$
Remainder: 0

16.
$$\begin{array}{r|rrrrrr} -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ & & -1 & 1 & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & -1 & 1 & 0 \end{array}$$
Quotient: $x^4 - x^3 + x^2 - x + 1$
Remainder: 0

17.
$$\begin{array}{r|rrrr} 2 & 4 & -3 & -8 & 4 \\ & & 8 & 10 & 4 \\ \hline & 4 & 5 & 2 & 8 \end{array}$$
Remainder = $8 \neq 0$. Therefore, $x - 2$ is not a factor of $4x^3 - 3x^2 - 8x + 4$.

18. $-3\overline{)\begin{array}{rrrr}-4 & 5 & 0 & 8\end{array}}$
$\quad\quad\begin{array}{rrrr}\quad & 12 & -51 & 153\end{array}$
$\quad\quad\overline{\begin{array}{rrrr}-4 & 17 & -51 & 161\end{array}}$

Remainder = $161 \neq 0$. Therefore, $x+3$ is not a factor of $-4x^3+5x^2+8$.

19. $2\overline{)\begin{array}{rrrrr}3 & -6 & 0 & -5 & 10\end{array}}$
$\quad\quad\begin{array}{rrrrr}\quad & 6 & 0 & 0 & -10\end{array}$
$\quad\quad\overline{\begin{array}{rrrrr}3 & 0 & 0 & -5 & 0\end{array}}$

Remainder = 0. Therefore, $x-2$ is a factor of $3x^4-6x^3-5x+10$.

20. $2\overline{)\begin{array}{rrrrr}4 & 0 & -15 & 0 & -4\end{array}}$
$\quad\quad\begin{array}{rrrrr}\quad & 8 & 16 & 2 & 4\end{array}$
$\quad\quad\overline{\begin{array}{rrrrr}4 & 8 & 1 & 2 & 0\end{array}}$

Remainder = 0. Therefore, $x-2$ is a factor of $4x^4-15x^2-4$.

21. $-3\overline{)\begin{array}{rrrrrrr}3 & 0 & 0 & 82 & 0 & 0 & 27\end{array}}$
$\quad\quad\begin{array}{rrrrrrr}\quad & -9 & 27 & -81 & -3 & 9 & -27\end{array}$
$\quad\quad\overline{\begin{array}{rrrrrrr}3 & -9 & 27 & 1 & -3 & 9 & 0\end{array}}$

Remainder = 0. Therefore, $x+3$ is a factor of $3x^6+82x^3+27$.

22. $-3\overline{)\begin{array}{rrrrrrr}2 & 0 & -18 & 0 & 1 & 0 & -9\end{array}}$
$\quad\quad\begin{array}{rrrrrrr}\quad & -6 & 18 & 0 & 0 & -3 & 9\end{array}$
$\quad\quad\overline{\begin{array}{rrrrrrr}2 & -6 & 0 & 0 & 1 & -3 & 0\end{array}}$

Remainder = 0. Therefore, $x+3$ is a factor of $2x^6-18x^4+x^2-9$.

23. $-4\overline{)\begin{array}{rrrrrrr}4 & 0 & -64 & 0 & 1 & 0 & -15\end{array}}$
$\quad\quad\begin{array}{rrrrrrr}\quad & -16 & 64 & 0 & 0 & -4 & 16\end{array}$
$\quad\quad\overline{\begin{array}{rrrrrrr}4 & -16 & 0 & 0 & 1 & -4 & 1\end{array}}$

Remainder = $1 \neq 0$. Therefore, $x+4$ is not a factor of $4x^6-64x^4+x^2-15$.

24. $-4\overline{)\begin{array}{rrrrrrr}1 & 0 & -16 & 0 & 1 & 0 & -16\end{array}}$
$\quad\quad\begin{array}{rrrrrrr}\quad & -4 & 16 & 0 & 0 & -4 & 16\end{array}$
$\quad\quad\overline{\begin{array}{rrrrrrr}1 & -4 & 0 & 0 & 1 & -4 & 0\end{array}}$

Remainder = 0. Therefore, $x+4$ is a factor $x^6-16x^4+x^2-16$.

25. $\frac{1}{2}\overline{)\begin{array}{rrrrr}2 & -1 & 0 & 2 & -1\end{array}}$
$\quad\quad\begin{array}{rrrrr}\quad & 1 & 0 & 0 & 1\end{array}$
$\quad\quad\overline{\begin{array}{rrrrr}2 & 0 & 0 & 2 & 0\end{array}}$

Remainder = 0; therefore $x-\frac{1}{2}$ is a factor of $2x^4-x^3+2x-1$.

26. $-\frac{1}{3}\overline{)\begin{array}{rrrrr}3 & 1 & 0 & -3 & 1\end{array}}$
$\quad\quad\begin{array}{rrrrr}\quad & -1 & 0 & 0 & 1\end{array}$
$\quad\quad\overline{\begin{array}{rrrrr}3 & 0 & 0 & -3 & 2\end{array}}$

Remainder = $2 \neq 0$; therefore $x+\frac{1}{3}$ is not a factor of $3x^4+x^3-3x+1$.

27. $-2\overline{)\begin{array}{rrrr}1 & -2 & 3 & 5\end{array}}$
$\quad\quad\begin{array}{rrrr}\quad & -2 & 8 & -22\end{array}$
$\quad\quad\overline{\begin{array}{rrrr}1 & -4 & 11 & -17\end{array}}$

$$\frac{x^3-2x^2+3x+5}{x+2} = x^2-4x+11+\frac{-17}{x+2}$$

$a+b+c+d = 1-4+11-17 = -9$

28. Answers will vary.

Section R.7

1. lowest terms

2. Least Common Multiple

3. True; $\dfrac{2x^3-4x}{x-2} = \dfrac{2x\left(x^2-2\right)}{x-2}$

4. False;
$2x^3+6x^2 = 2x^2(x+3)$
$6x^4+4x^3 = 2x^3(3x+2)$
$LCM = 2x^3(x+3)(3x+2)$

5. $\dfrac{3x+9}{x^2-9} = \dfrac{3(x+3)}{(x-3)(x+3)} = \dfrac{3}{x-3}$

6. $\dfrac{4x^2+8x}{12x+24}=\dfrac{4x(x+2)}{12(x+2)}=\dfrac{x}{3}$

7. $\dfrac{x^2-2x}{3x-6}=\dfrac{x(x-2)}{3(x-2)}=\dfrac{x}{3}$

8. $\dfrac{15x^2+24x}{3x^2}=\dfrac{3x(5x+8)}{3x^2}=\dfrac{5x+8}{x}$

9. $\dfrac{24x^2}{12x^2-6x}=\dfrac{24x^2}{6x(2x-1)}=\dfrac{4x}{2x-1}$

10. $\dfrac{x^2+4x+4}{x^2-4}=\dfrac{(x+2)(x+2)}{(x-2)(x+2)}=\dfrac{x+2}{x-2}$

11. $\dfrac{y^2-25}{2y^2-8y-10}=\dfrac{(y+5)(y-5)}{2(y^2-4y-5)}$
$=\dfrac{(y+5)(y-5)}{2(y-5)(y+1)}$
$=\dfrac{y+5}{2(y+1)}$

12. $\dfrac{3y^2-y-2}{3y^2+5y+2}=\dfrac{(3y+2)(y-1)}{(3y+2)(y+1)}=\dfrac{y-1}{y+1}$

13. $\dfrac{x^2+4x-5}{x^2-2x+1}=\dfrac{(x+5)(x-1)}{(x-1)(x-1)}=\dfrac{x+5}{x-1}$

14. $\dfrac{x-x^2}{x^2+x-2}=\dfrac{-x(x-1)}{(x+2)(x-1)}=\dfrac{-x}{x+2}=-\dfrac{x}{x+2}$

15. $\dfrac{x^2+5x-14}{2-x}=\dfrac{(x+7)(x-2)}{2-x}$
$=\dfrac{(x+7)(x-2)}{-1(x-2)}$
$=-(x+7)$
$=-x-7$

16. $\dfrac{2x^2+5x-3}{1-2x}=\dfrac{(2x-1)(x+3)}{-1(2x-1)}=-(x+3)=-x-3$

17. $\dfrac{3x+6}{5x^2}\cdot\dfrac{x}{x^2-4}=\dfrac{3(x+2)}{5x^2}\cdot\dfrac{x}{(x-2)(x+2)}$
$=\dfrac{3}{5x(x-2)}$

18. $\dfrac{3}{2x}\cdot\dfrac{x^2}{6x+10}=\dfrac{3}{2}\cdot\dfrac{x}{2(3x+5)}=\dfrac{3x}{4(3x+5)}$

19. $\dfrac{4x^2}{x^2-16}\cdot\dfrac{x^3-64}{2x}$
$=\dfrac{4x^2}{(x-4)(x+4)}\cdot\dfrac{(x-4)\left(x^2+4x+16\right)}{2x}$
$=\dfrac{2x\cdot 2x(x-4)\left(x^2+4x+16\right)}{2x(x-4)(x+4)}$
$=\dfrac{2x\left(x^2+4x+16\right)}{x+4}$

20. $\dfrac{12}{x^2+x}\cdot\dfrac{x^3+1}{4x-2}=\dfrac{12}{x(x+1)}\cdot\dfrac{(x+1)(x^2-x+1)}{2(2x-1)}$
$=\dfrac{2\cdot 6(x+1)\left(x^2-x+1\right)}{2x(x+1)(2x-1)}$
$=\dfrac{6\left(x^2-x+1\right)}{x(2x-1)}$

21. $\dfrac{4x-8}{-3x}\cdot\dfrac{12}{12-6x}=\dfrac{4(x-2)}{-3x}\cdot\dfrac{12}{6(2-x)}$
$=\dfrac{4(x-2)}{-3x}\cdot\dfrac{2}{(-1)(x-2)}$
$=\dfrac{8}{3x}$

22. $\dfrac{6x-27}{5x}\cdot\dfrac{2}{4x-18}=\dfrac{3(2x-9)}{5x}\cdot\dfrac{2}{2(2x-9)}=\dfrac{3}{5x}$

23. $\dfrac{x^2-3x-10}{x^2+2x-35}\cdot\dfrac{x^2+4x-21}{x^2+9x+14}$
$=\dfrac{(x-5)(x+2)}{(x+7)(x-5)}\cdot\dfrac{(x+7)(x-3)}{(x+7)(x+2)}$
$=\dfrac{x-3}{x+7}$

24. $\dfrac{x^2+x-6}{x^2+4x-5}\cdot\dfrac{x^2-25}{x^2+2x-15}$
$=\dfrac{(x-2)(x+3)}{(x+5)(x-1)}\cdot\dfrac{(x+5)(x-5)}{(x+5)(x-3)}$
$=\dfrac{(x-2)(x+3)(x-5)}{(x+5)(x-1)(x-3)}$

25. $$\frac{\frac{6x}{x^2-4}}{\frac{3x-9}{2x+4}} = \frac{6x}{x^2-4}\cdot\frac{2x+4}{3x-9}$$
$$= \frac{6x}{(x-2)(x+2)}\cdot\frac{2(x+2)}{3(x-3)}$$
$$= \frac{4x}{(x-2)(x-3)}$$

26. $$\frac{\frac{12x}{5x+20}}{\frac{4x^2}{x^2-16}} = \frac{12x}{5x+20}\cdot\frac{x^2-16}{4x^2}$$
$$= \frac{12x}{5(x+4)}\cdot\frac{(x+4)(x-4)}{4x^2}$$
$$= \frac{3(x-4)}{5x}$$

27. $$\frac{\frac{8x}{x^2-1}}{\frac{10x}{x+1}} = \frac{8x}{x^2-1}\cdot\frac{x+1}{10x}$$
$$= \frac{8x}{(x-1)(x+1)}\cdot\frac{x+1}{10x}$$
$$= \frac{4}{5(x-1)}$$

28. $$\frac{\frac{x-2}{4x}}{\frac{x^2-4x+4}{12x}} = \frac{x-2}{4x}\cdot\frac{12x}{x^2-4x+4}$$
$$= \frac{x-2}{4x}\cdot\frac{12x}{(x-2)(x-2)}$$
$$= \frac{3}{x-2}$$

29. $$\frac{\frac{4-x}{4+x}}{\frac{4x}{x^2-16}} = \frac{4-x}{4+x}\cdot\frac{x^2-16}{4x}$$
$$= \frac{4-x}{4+x}\cdot\frac{(x+4)(x-4)}{4x}$$
$$= \frac{(4-x)(x-4)}{4x}$$
$$= -\frac{(x-4)^2}{4x}$$

30. $$\frac{\frac{3+x}{3-x}}{\frac{x^2-9}{9x^3}} = \frac{3+x}{3-x}\cdot\frac{9x^3}{x^2-9}$$
$$= \frac{3+x}{3-x}\cdot\frac{9x^3}{(x+3)(x-3)}$$
$$= \frac{9x^3}{(3-x)(x-3)}$$
$$= \frac{9x^3}{-(x-3)^2}$$
$$= -\frac{9x^3}{(x-3)^2}$$

31. $$\frac{\frac{x^2+7x+12}{x^2-7x+12}}{\frac{x^2+x-12}{x^2-x-12}} = \frac{x^2+7x+12}{x^2-7x+12}\cdot\frac{x^2-x-12}{x^2+x-12}$$
$$= \frac{(x+3)(x+4)}{(x-3)(x-4)}\cdot\frac{(x-4)(x+3)}{(x+4)(x-3)}$$
$$= \frac{(x+3)^2}{(x-3)^2}$$

32. $$\frac{\frac{x^2+7x+6}{x^2+x-6}}{\frac{x^2+5x-6}{x^2+5x+6}} = \frac{x^2+7x+6}{x^2+x-6}\cdot\frac{x^2+5x+6}{x^2+5x-6}$$
$$= \frac{(x+6)(x+1)}{(x+3)(x-2)}\cdot\frac{(x+2)(x+3)}{(x+6)(x-1)}$$
$$= \frac{(x+1)(x+2)}{(x-2)(x-1)}$$

33. $$\frac{\frac{2x^2-x-28}{3x^2-x-2}}{\frac{4x^2+16x+7}{3x^2+11x+6}} = \frac{2x^2-x-28}{3x^2-x-2}\cdot\frac{3x^2+11x+6}{4x^2+16x+7}$$
$$= \frac{(2x+7)(x-4)}{(3x+2)(x-1)}\cdot\frac{(3x+2)(x+3)}{(2x+7)(2x+1)}$$
$$= \frac{(x-4)(x+3)}{(x-1)(2x+1)}$$

34. $$\frac{\dfrac{9x^2+3x-2}{12x^2+5x-2}}{\dfrac{9x^2-6x+1}{8x^2-10x-3}} = \frac{9x^2+3x-2}{12x^2+5x-2}\cdot\frac{8x^2-10x-3}{9x^2-6x+1}$$
$$= \frac{(3x+2)(3x-1)}{(3x+2)(4x-1)}\cdot\frac{(4x+1)(2x-3)}{(3x-1)(3x-1)}$$
$$= \frac{(4x+1)(2x-3)}{(4x-1)(3x-1)}$$

35. $$\frac{x}{2}+\frac{5}{2} = \frac{x+5}{2}$$

36. $$\frac{3}{x}-\frac{6}{x} = \frac{3-6}{x} = \frac{-3}{x} = -\frac{3}{x}$$

37. $$\frac{x^2}{2x-3}-\frac{4}{2x-3} = \frac{x^2-4}{2x-3} = \frac{(x+2)(x-2)}{2x-3}$$

38. $$\frac{3x^2}{2x-1}-\frac{9}{2x-1} = \frac{3x^2-9}{2x-1} = \frac{3\left(x^2-3\right)}{2x-1}$$

39. $$\frac{x+1}{x-3}+\frac{2x-3}{x-3} = \frac{x+1+2x-3}{x-3} = \frac{3x-2}{x-3}$$

40. $$\frac{2x-5}{3x+2}+\frac{x+4}{3x+2} = \frac{2x-5+x+4}{3x+2} = \frac{3x-1}{3x+2}$$

41. $$\frac{3x+5}{2x-1}-\frac{2x-4}{2x-1} = \frac{(3x+5)-(2x-4)}{2x-1}$$
$$= \frac{3x+5-2x+4}{2x-1}$$
$$= \frac{x+9}{2x-1}$$

42. $$\frac{5x-4}{3x+4}-\frac{x+1}{3x+4} = \frac{(5x-4)-(x+1)}{3x+4}$$
$$= \frac{5x-4-x-1}{3x+4}$$
$$= \frac{4x-5}{3x+4}$$

43. $$\frac{4}{x-2}+\frac{x}{2-x} = \frac{4}{x-2}-\frac{x}{x-2} = \frac{4-x}{x-2}$$

44. $$\frac{6}{x-1}-\frac{x}{1-x} = \frac{6}{x-1}+\frac{x}{x-1} = \frac{x+6}{x-1}$$

45. $$\frac{4}{x-1}-\frac{2}{x+2} = \frac{4(x+2)}{(x-1)(x+2)}-\frac{2(x-1)}{(x+2)(x-1)}$$
$$= \frac{4x+8-2x+2}{(x+2)(x-1)}$$
$$= \frac{2x+10}{(x+2)(x-1)}$$
$$= \frac{2(x+5)}{(x+2)(x-1)}$$

46. $$\frac{2}{x+5}-\frac{5}{x-5} = \frac{2(x-5)}{(x+5)(x-5)}-\frac{5(x+5)}{(x+5)(x-5)}$$
$$= \frac{2x-10-5x-25}{(x+5)(x-5)}$$
$$= \frac{-3x-35}{(x+5)(x-5)}$$
$$= -\frac{3x+35}{(x+5)(x-5)}$$

47. $$\frac{x}{x+1}+\frac{2x-3}{x-1} = \frac{x(x-1)}{(x+1)(x-1)}+\frac{(2x-3)(x+1)}{(x-1)(x+1)}$$
$$= \frac{x^2-x+2x^2-x-3}{(x-1)(x+1)}$$
$$= \frac{3x^2-2x-3}{(x-1)(x+1)}$$

48. $$\frac{3x}{x-4}+\frac{2x}{x+3} = \frac{3x(x+3)}{(x-4)(x+3)}+\frac{2x(x-4)}{(x-4)(x+3)}$$
$$= \frac{3x^2+9x+2x^2-8x}{(x-4)(x+3)}$$
$$= \frac{5x^2+x}{(x-4)(x+3)}$$
$$= \frac{x(5x+1)}{(x-4)(x+3)}$$

49. $$\frac{x-3}{x+2}-\frac{x+4}{x-2} = \frac{(x-3)(x-2)}{(x+2)(x-2)}-\frac{(x+4)(x+2)}{(x-2)(x+2)}$$
$$= \frac{x^2-5x+6-(x^2+6x+8)}{(x+2)(x-2)}$$
$$= \frac{x^2-5x+6-x^2-6x-8}{(x+2)(x-2)}$$
$$= \frac{-11x-2}{(x+2)(x-2)} \text{ or } \frac{-(11x+2)}{(x+2)(x-2)}$$

50. $\dfrac{2x-3}{x-1}-\dfrac{2x+1}{x+1}=\dfrac{(2x-3)(x+1)}{(x-1)(x+1)}-\dfrac{(2x+1)(x-1)}{(x+1)(x-1)}$

$=\dfrac{2x^2-x-3-(2x^2-x-1)}{(x+1)(x-1)}$

$=\dfrac{2x^2-x-3-2x^2+x+1}{(x+1)(x-1)}$

$=\dfrac{-2}{(x+1)(x-1)}$

$=-\dfrac{2}{(x+1)(x-1)}$

51. $\dfrac{x}{x^2-4}+\dfrac{1}{x}=\dfrac{x^2+x^2-4}{x\left(x^2-4\right)}$

$=\dfrac{2x^2-4}{x\left(x^2-4\right)}$

$=\dfrac{2\left(x^2-2\right)}{x(x-2)(x+2)}$

52. $\dfrac{x-1}{x^3}+\dfrac{x}{x^2+1}=\dfrac{(x-1)\left(x^2+1\right)+x^4}{x^3\left(x^2+1\right)}$

$=\dfrac{x^3-x^2+x-1+x^4}{x^3\left(x^2+1\right)}$

$=\dfrac{x^4+x^3-x^2+x-1}{x^3\left(x^2+1\right)}$

53. $x^2-4=(x+2)(x-2)$

$x^2-x-2=(x+1)(x-2)$

Therefore, LCM $=(x+2)(x-2)(x+1)$.

54. $x^2-x-12=(x+3)(x-4)$

$x^2-8x+16=(x-4)(x-4)$

Therefore, LCM $=(x+3)(x-4)^2$.

55. $x^3-x=x\left(x^2-1\right)=x(x+1)(x-1)$

$x^2-x=x(x-1)$

Therefore, LCM $=x(x+1)(x-1)$.

56. $3x^2-27=3\left(x^2-9\right)=3(x+3)(x-3)$

$2x^2-x-15=(2x+5)(x-3)$

Therefore, LCM $=3(2x+5)(x-3)(x+3)$.

57. $4x^3-4x^2+x=x\left(4x^2-4x+1\right)$

$=x(2x-1)(2x-1)$

$2x^3-x^2=x^2(2x-1)$

x^3

Therefore, LCM $=x^3(2x-1)^2$.

58. $x-3$

$x^2+3x=x(x+3)$

$x^3-9x=x\left(x^2-9\right)=x(x+3)(x-3)$

Therefore, LCM $=x(x+3)(x-3)$.

59. $x^3-x=x\left(x^2-1\right)=x(x+1)(x-1)$

$x^3-2x^2+x=x\left(x^2-2x+1\right)=x(x-1)^2$

$x^3-1=(x-1)\left(x^2+x+1\right)$

Therefore, LCM $=x(x+1)(x-1)^2\left(x^2+x+1\right)$.

60. $x^2+4x+4=(x+2)^2$

$x^3+2x^2=x^2(x+2)$

$(x+2)^3$

Therefore, LCM $=x^2(x+2)^3$.

61. $\dfrac{x}{x^2-7x+6}-\dfrac{x}{x^2-2x-24}$

$=\dfrac{x}{(x-6)(x-1)}-\dfrac{x}{(x-6)(x+4)}$

$=\dfrac{x(x+4)}{(x-6)(x-1)(x+4)}-\dfrac{x(x-1)}{(x-6)(x+4)(x-1)}$

$=\dfrac{x^2+4x-x^2+x}{(x-6)(x+4)(x-1)}=\dfrac{5x}{(x-6)(x+4)(x-1)}$

62. $\dfrac{x}{x-3}-\dfrac{x+1}{x^2+5x-24}$

$=\dfrac{x}{(x-3)}-\dfrac{x+1}{(x-3)(x+8)}$

$=\dfrac{x(x+8)}{(x-3)(x+8)}-\dfrac{x+1}{(x-3)(x+8)}$

$=\dfrac{x^2+8x-x-1}{(x-3)(x+8)}=\dfrac{x^2+7x-1}{(x-3)(x+8)}$

63. $\dfrac{4x}{x^2-4}-\dfrac{2}{x^2+x-6}$

$=\dfrac{4x}{(x-2)(x+2)}-\dfrac{2}{(x+3)(x-2)}$

$=\dfrac{4x(x+3)}{(x-2)(x+2)(x+3)}-\dfrac{2(x+2)}{(x+3)(x-2)(x+2)}$

$=\dfrac{4x^2+12x-2x-4}{(x-2)(x+2)(x+3)}$

$=\dfrac{4x^2+10x-4}{(x-2)(x+2)(x+3)}$

$=\dfrac{2(2x^2+5x-2)}{(x-2)(x+2)(x+3)}$

64. $\dfrac{3x}{x-1}-\dfrac{x-4}{x^2-2x+1}=\dfrac{3x}{(x-1)}-\dfrac{x-4}{(x-1)^2}$

$=\dfrac{3x(x-1)}{(x-1)(x-1)}-\dfrac{x-4}{(x-1)^2}$

$=\dfrac{3x^2-3x-x+4}{(x-1)^2}$

$=\dfrac{3x^2-4x+4}{(x-1)^2}$

65. $\dfrac{3}{(x-1)^2(x+1)}+\dfrac{2}{(x-1)(x+1)^2}$

$=\dfrac{3(x+1)+2(x-1)}{(x-1)^2(x+1)^2}$

$=\dfrac{3x+3+2x-2}{(x-1)^2(x+1)^2}$

$=\dfrac{5x+1}{(x-1)^2(x+1)^2}$

66. $\dfrac{2}{(x+2)^2(x-1)}-\dfrac{6}{(x+2)(x-1)^2}$

$=\dfrac{2(x-1)-6(x+2)}{(x+2)^2(x-1)^2}$

$=\dfrac{2x-2-6x-12}{(x+2)^2(x-1)^2}$

$=\dfrac{-4x-14}{(x+2)^2(x-1)^2}$

$=\dfrac{-2(2x+7)}{(x+2)^2(x-1)^2}$

67. $\dfrac{x+4}{x^2-x-2}-\dfrac{2x+3}{x^2+2x-8}$

$=\dfrac{x+4}{(x-2)(x+1)}-\dfrac{2x+3}{(x+4)(x-2)}$

$=\dfrac{(x+4)(x+4)}{(x-2)(x+1)(x+4)}-\dfrac{(2x+3)(x+1)}{(x+4)(x-2)(x+1)}$

$=\dfrac{x^2+8x+16-(2x^2+5x+3)}{(x-2)(x+1)(x+4)}$

$=\dfrac{-x^2+3x+13}{(x-2)(x+1)(x+4)}$

68. $\dfrac{2x-3}{x^2+8x+7}-\dfrac{x-2}{(x+1)^2}$

$=\dfrac{2x-3}{(x+1)(x+7)}-\dfrac{x-2}{(x+1)^2}$

$=\dfrac{(2x-3)(x+1)}{(x+1)(x+7)(x+1)}-\dfrac{(x-2)(x+7)}{(x+1)^2(x+7)}$

$=\dfrac{2x^2-x-3-(x^2+5x-14)}{(x+1)^2(x+7)}$

$=\dfrac{x^2-6x+11}{(x+1)^2(x+7)}$

69. $\dfrac{1}{x}-\dfrac{2}{x^2+x}+\dfrac{3}{x^3-x^2}$

$=\dfrac{1}{x}-\dfrac{2}{x(x+1)}+\dfrac{3}{x^2(x-1)}$

$=\dfrac{x(x+1)(x-1)-2x(x-1)+3(x+1)}{x^2(x+1)(x-1)}$

$=\dfrac{x(x^2-1)-2x^2+2x+3x+3}{x^2(x+1)(x-1)}$

$=\dfrac{x^3-x-2x^2+5x+3}{x^2(x+1)(x-1)}$

$=\dfrac{x^3-2x^2+4x+3}{x^2(x+1)(x-1)}$

70. $\dfrac{x}{(x-1)^2}+\dfrac{2}{x}-\dfrac{x+1}{x^3-x^2}$

$=\dfrac{x}{(x-1)^2}+\dfrac{2}{x}-\dfrac{x+1}{x^2(x-1)}$

$=\dfrac{x^3+2x(x-1)^2-(x+1)(x-1)}{x^2(x-1)^2}$

$=\dfrac{x^3+2x(x^2-2x+1)-(x^2-1)}{x^2(x-1)^2}$

$=\dfrac{x^3+2x^3-4x^2+2x-x^2+1}{x^2(x-1)^2}$

$=\dfrac{3x^3-5x^2+2x+1}{x^2(x-1)^2}$

71. $\dfrac{1}{h}\left(\dfrac{1}{x+h}-\dfrac{1}{x}\right)=\dfrac{1}{h}\left(\dfrac{1\cdot x}{(x+h)x}-\dfrac{1(x+h)}{x(x+h)}\right)$

$=\dfrac{1}{h}\left(\dfrac{x-x-h}{x(x+h)}\right)$

$=\dfrac{-h}{hx(x+h)}$

$=\dfrac{-1}{x(x+h)}$

72. $\dfrac{1}{h}\left(\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right)$

$=\dfrac{1}{h}\left(\dfrac{1\cdot x^2}{(x+h)^2x^2}-\dfrac{1(x+h)^2}{x^2(x+h)^2}\right)$

$=\dfrac{1}{h}\left(\dfrac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}\right)$

$=\dfrac{-2xh-h^2}{hx^2(x+h)^2}$

$=\dfrac{h(-2x-h)}{hx^2(x+h)^2}$

$=\dfrac{-2x-h}{x^2(x+h)^2}$

$=-\dfrac{2x+h}{x^2(x+h)^2}$

73. $\dfrac{1+\dfrac{1}{x}}{1-\dfrac{1}{x}}=\dfrac{\left(\dfrac{x}{x}+\dfrac{1}{x}\right)}{\left(\dfrac{x}{x}-\dfrac{1}{x}\right)}=\dfrac{\left(\dfrac{x+1}{x}\right)}{\left(\dfrac{x-1}{x}\right)}=\dfrac{x+1}{x}\cdot\dfrac{x}{x-1}=\dfrac{x+1}{x-1}$

74. $\dfrac{4+\dfrac{1}{x^2}}{3-\dfrac{1}{x^2}}=\dfrac{\left(\dfrac{4x^2}{x^2}+\dfrac{1}{x^2}\right)}{\left(\dfrac{3x^2}{x^2}-\dfrac{1}{x^2}\right)}=\dfrac{\left(\dfrac{4x^2+1}{x^2}\right)}{\left(\dfrac{3x^2-1}{x^2}\right)}$

$=\dfrac{4x^2+1}{x^2}\cdot\dfrac{x^2}{3x^2-1}$

$=\dfrac{4x^2+1}{3x^2-1}$

75. $\dfrac{2-\dfrac{x+1}{x}}{3+\dfrac{x-1}{x+1}}=\dfrac{\dfrac{2x}{x}-\dfrac{x+1}{x}}{\dfrac{3(x+1)}{x+1}+\dfrac{x-1}{x+1}}=\dfrac{\dfrac{2x-x-1}{x}}{\dfrac{3x+3+x-1}{x+1}}$

$=\dfrac{\dfrac{x-1}{x}}{\dfrac{4x+2}{x+1}}=\dfrac{x-1}{x}\cdot\dfrac{x+1}{2(2x+1)}$

$=\dfrac{(x-1)(x+1)}{2x(2x+1)}$

76. $\dfrac{1-\dfrac{x}{x+1}}{2-\dfrac{x-1}{x}}=\dfrac{\left(\dfrac{x+1}{x+1}-\dfrac{x}{x+1}\right)}{\left(\dfrac{2x}{x}-\dfrac{x-1}{x}\right)}=\dfrac{\left(\dfrac{1}{x+1}\right)}{\left(\dfrac{x+1}{x}\right)}$

$=\dfrac{1}{x+1}\cdot\dfrac{x}{x+1}$

$=\dfrac{x}{(x+1)^2}$

77. $\dfrac{\dfrac{x+4}{x-2}-\dfrac{x-3}{x+1}}{x+1}$

$=\dfrac{\left(\dfrac{(x+4)(x+1)}{(x-2)(x+1)}-\dfrac{(x-3)(x-2)}{(x+1)(x-2)}\right)}{x+1}$

$=\dfrac{\left(\dfrac{x^2+5x+4-(x^2-5x+6)}{(x-2)(x+1)}\right)}{x+1}$

$=\dfrac{10x-2}{(x-2)(x+1)}\cdot\dfrac{1}{x+1}$

$=\dfrac{2(5x-1)}{(x-2)(x+1)^2}$

78. $\dfrac{\dfrac{x-2}{x+1}-\dfrac{x}{x-2}}{x+3}$

$$=\frac{\left(\dfrac{(x-2)(x-2)}{(x+1)(x-2)}-\dfrac{x(x+1)}{(x-2)(x+1)}\right)}{x+3}$$

$$=\frac{\left(\dfrac{x^2-4x+4-(x^2+x)}{(x-2)(x+1)}\right)}{x+3}$$

$$=\frac{-5x+4}{(x-2)(x+1)}\cdot\frac{1}{x+3}$$

$$=\frac{-5x+4}{(x-2)(x+1)(x+3)}$$

$$=\frac{-(5x-4)}{(x-2)(x+1)(x+3)}$$

79. $\dfrac{\dfrac{x-2}{x+2}+\dfrac{x-1}{x+1}}{\dfrac{x}{x+1}-\dfrac{2x-3}{x}}$

$$=\frac{\left(\dfrac{(x-2)(x+1)}{(x+2)(x+1)}+\dfrac{(x-1)(x+2)}{(x+1)(x+2)}\right)}{\left(\dfrac{x^2}{(x+1)(x)}-\dfrac{(2x-3)(x+1)}{x(x+1)}\right)}$$

$$=\frac{\left(\dfrac{x^2-x-2+x^2+x-2}{(x+2)(x+1)}\right)}{\left(\dfrac{x^2-(2x^2-x-3)}{x(x+1)}\right)}$$

$$=\frac{\left(\dfrac{2x^2-4}{(x+2)(x+1)}\right)}{\left(\dfrac{-x^2+x+3}{x(x+1)}\right)}$$

$$=\frac{2(x^2-2)}{(x+2)(x+1)}\cdot\frac{x(x+1)}{-(x^2-x-3)}$$

$$=\frac{2x(x^2-2)}{-(x+2)(x^2-x-3)}$$

$$=\frac{-2x(x^2-2)}{(x+2)(x^2-x-3)}$$

80. $\dfrac{\dfrac{2x+5}{x}-\dfrac{x}{x-3}}{\dfrac{x^2}{x-3}-\dfrac{(x+1)^2}{x+3}}$

$$=\frac{\left(\dfrac{(2x+5)(x-3)}{x(x-3)}-\dfrac{x(x)}{x(x-3)}\right)}{\left(\dfrac{x^2(x+3)}{(x-3)(x+3)}-\dfrac{(x-3)(x+1)^2}{(x-3)(x+3)}\right)}$$

$$=\frac{\left(\dfrac{2x^2-x-15-x^2}{x(x-3)}\right)}{\left(\dfrac{x^3+3x^2-(x^3-x^2-5x-3)}{(x-3)(x+3)}\right)}$$

$$=\frac{\left(\dfrac{x^2-x-15}{x(x-3)}\right)}{\left(\dfrac{4x^2+5x+3}{(x-3)(x+3)}\right)}$$

$$=\frac{x^2-x-15}{x(x-3)}\cdot\frac{(x-3)(x+3)}{4x^2+5x+3}$$

$$=\frac{(x^2-x-15)(x+3)}{x(4x^2+5x+3)}$$

81. $1-\dfrac{1}{1-\dfrac{1}{x}}=1-\dfrac{1}{\dfrac{x-1}{x}}$

$$=1-\frac{x}{x-1}$$

$$=\frac{x-1-x}{x-1}$$

$$=\frac{-1}{x-1}$$

82. $1-\dfrac{1}{1-\dfrac{1}{1-x}}=1-\dfrac{1}{\dfrac{1-x-1}{1-x}}=1-\dfrac{1}{\dfrac{-x}{1-x}}$

$$=1-\frac{1-x}{-x}=1+\frac{1-x}{x}$$

$$=\frac{x+1-x}{x}$$

$$=\frac{1}{x}$$

83. $$\frac{2(x-1)^{-1}+3}{3(x-1)^{-1}+2}=\frac{\frac{2}{x-1}+3}{\frac{3}{x-1}+2}=\frac{\frac{2}{x-1}+\frac{3(x-1)}{x-1}}{\frac{3}{x-1}+\frac{2(x-1)}{x-1}}$$
$$=\frac{\frac{2+3(x-1)}{x-1}}{\frac{3+2(x-1)}{x-1}}$$
$$=\frac{2+3(x-1)}{x-1}\cdot\frac{x-1}{3+2(x-1)}$$
$$=\frac{2+3(x-1)}{3+2(x-1)}=\frac{2+3x-3}{3+2x-2}$$
$$=\frac{3x-1}{2x+1}$$

84. $$\frac{4(x+2)^{-1}-3}{3(x+2)^{-1}-1}=\frac{\frac{4}{x+2}-3}{\frac{3}{x+2}-1}=\frac{\frac{4}{x+2}-\frac{3(x+2)}{x+2}}{\frac{3}{x+2}-\frac{1(x+2)}{x+2}}$$
$$=\frac{\frac{4-3(x+2)}{x+2}}{\frac{3-(x+2)}{x+2}}$$
$$=\frac{4-3(x+2)}{x+2}\cdot\frac{x+2}{3-(x+2)}$$
$$=\frac{4-3(x+2)}{3-(x+2)}=\frac{4-3x-6}{3-x-2}$$
$$=\frac{-3x-2}{-x+1}=\frac{3x+2}{x-1}$$

85. $$\frac{(2x+3)\cdot 3-(3x-5)\cdot 2}{(3x-5)^2}=\frac{6x+9-6x+10}{(3x-5)^2}$$
$$=\frac{19}{(3x-5)^2}$$

86. $$\frac{(4x+1)\cdot 5-(5x-2)\cdot 4}{(5x-2)^2}=\frac{20x+5-20x+8}{(5x-2)^2}$$
$$=\frac{13}{(5x-2)^2}$$

87. $$\frac{x\cdot 2x-(x^2+1)\cdot 1}{(x^2+1)^2}=\frac{2x^2-x^2-1}{(x^2+1)^2}$$
$$=\frac{x^2-1}{(x^2+1)^2}$$
$$=\frac{(x-1)(x+1)}{(x^2+1)^2}$$

88. $$\frac{x\cdot 2x-(x^2-4)\cdot 1}{(x^2-4)^2}=\frac{2x^2-x^2+4}{(x^2-4)^2}=\frac{x^2+4}{(x^2-4)^2}$$
$$=\frac{x^2+4}{(x+2)^2(x-2)^2}$$

89. $$\frac{(3x+1)\cdot 2x-x^2\cdot 3}{(3x+1)^2}=\frac{6x^2+2x-3x^2}{(3x+1)^2}$$
$$=\frac{3x^2+2x}{(3x+1)^2}$$
$$=\frac{x(3x+2)}{(3x+1)^2}$$

90. $$\frac{(2x-5)\cdot 3x^2-x^3\cdot 2}{(2x-5)^2}=\frac{6x^3-15x^2-2x^3}{(2x-5)^2}$$
$$=\frac{4x^3-15x^2}{(2x-5)^2}$$
$$=\frac{x^2(4x-15)}{(2x-5)^2}$$

91. $$\frac{(x^2+1)\cdot 3-(3x+4)\cdot 2x}{(x^2+1)^2}=\frac{3x^2+3-6x^2-8x}{(x^2+1)^2}$$
$$=\frac{-3x^2-8x+3}{(x^2+1)^2}$$
$$=\frac{-(3x^2+8x-3)}{(x^2+1)^2}$$
$$=-\frac{(3x-1)(x+3)}{(x^2+1)^2}$$

92. $$\frac{\left(x^2+9\right)\cdot 2-(2x-5)\cdot 2x}{\left(x^2+9\right)^2}=\frac{2x^2+18-4x^2+10x}{\left(x^2+9\right)^2}$$
$$=\frac{-2x^2+10x+18}{\left(x^2+9\right)^2}$$
$$=\frac{-2\left(x^2-5x-9\right)}{\left(x^2+9\right)^2}$$

93. $$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$
$$\frac{1}{f}=(n-1)\left(\frac{R_2+R_1}{R_1\cdot R_2}\right)$$
$$\frac{R_1\cdot R_2}{f}=(n-1)\left(R_2+R_1\right)$$
$$\frac{f}{R_1\cdot R_2}=\frac{1}{(n-1)\left(R_2+R_1\right)}$$
$$f=\frac{R_1\cdot R_2}{(n-1)\left(R_2+R_1\right)}$$
$$f=\frac{0.1(0.2)}{(1.5-1)(0.2+0.1)}$$
$$=\frac{0.02}{0.5(0.3)}=\frac{0.02}{0.15}=\frac{2}{15}\text{ meters}$$

94. $$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{R_2R_3+R_1R_3+R_1R_2}{R_1R_2R_3}$$
$$R=\frac{R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}$$
$$=\frac{5\cdot 4\cdot 10}{4\cdot 10+5\cdot 10+5\cdot 4}$$
$$=\frac{200}{110}=\frac{20}{11}\text{ ohms}$$

95. $$1+\frac{1}{x}=\frac{x+1}{x}\Rightarrow a=1,b=1,c=0$$
$$1+\frac{1}{1+\frac{1}{x}}=1+\frac{1}{\left(\frac{x+1}{x}\right)}=1+\frac{x}{x+1}$$
$$=\frac{x+1+x}{x+1}=\frac{2x+1}{x+1}$$
$$\Rightarrow a=2,b=1,c=1$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}=1+\frac{1}{\left(\frac{2x+1}{x+1}\right)}=1+\frac{x+1}{2x+1}$$
$$=\frac{2x+1+x+1}{2x+1}=\frac{3x+2}{2x+1}$$
$$\Rightarrow a=3,b=2,c=1$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}=1+\frac{1}{\left(\frac{3x+2}{2x+1}\right)}=1+\frac{2x+1}{3x+2}$$
$$=\frac{3x+2+2x+1}{3x+2}=\frac{5x+3}{3x+2}$$
$$\Rightarrow a=5,b=3,c=2$$

If we continue this process, the values of a, b and c produce the following sequences:

$a: 1,2,3,5,8,13,21,....$

$b: 1,1,2,3,5,8,13,21,.....$

$c: 0,1,1,2,3,5,8,13,21,.....$

In each case we have a *Fibonacci Sequence*, where the next value in the list is obtained from the sum of the previous 2 values in the list.

96. Answers will vary.

97. Answers will vary.

Section R.8

1. 9; –9

2. 4; $|-4| = 4$

3. index

4. True

5. cube root

6. False; $\sqrt[4]{(-3)^4} = |-3| = 3$

7. $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$

8. $\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

9. $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

10. $\sqrt[3]{-1} = \sqrt[3]{(-1)^3} = -1$

11. $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

12. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = 3\sqrt[3]{2}$

13. $\sqrt[3]{-8x^4} = \sqrt[3]{-8x^3 \cdot x} = -2x\sqrt[3]{x}$

14. $\sqrt[4]{48x^5} = \sqrt[4]{16x^4 \cdot 3x} = 2x\sqrt[4]{3x}$

15. $\sqrt[4]{x^{12}y^8} = \sqrt[4]{\left(x^3\right)^4\left(y^2\right)^4} = x^3y^2$

16. $\sqrt[5]{x^{10}y^5} = \sqrt[5]{\left(x^2\right)^5 y^5} = x^2y$

17. $\sqrt[4]{\dfrac{x^9y^7}{xy^3}} = \sqrt[4]{x^8y^4} = x^2y$

18. $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}} = \sqrt[3]{\dfrac{1}{27x^3}} = \dfrac{\sqrt[3]{1}}{\sqrt[3]{27x^3}} = \dfrac{1}{3x}$

19. $\sqrt{36x} = 6\sqrt{x}$

20. $\sqrt{9x^5} = 3\sqrt{x^4 \cdot x} = 3x^2\sqrt{x}$

21. $\sqrt{3x^2}\sqrt{12x} = \sqrt{36x^2 \cdot x} = 6x\sqrt{x}$

22. $\sqrt{5x}\sqrt{20x^3} = \sqrt{100x^4} = 10x^2$

23. $\left(\sqrt{5}\sqrt[3]{9}\right)^2 = \left(\sqrt{5}\right)^2\left(\sqrt[3]{9}\right)^2$

$= 5 \cdot \sqrt[3]{9^2} = 5\sqrt[3]{81} = 5 \cdot 3\sqrt[3]{3} = 15\sqrt[3]{3}$

24. $\left(\sqrt[3]{3}\sqrt{10}\right)^4 = \left(\sqrt[3]{3}\right)^4\left(\sqrt{10}\right)^4$

$= \sqrt[3]{3^4} \cdot 10^2 = 3\sqrt[3]{3} \cdot 100 = 300\sqrt[3]{3}$

25. $\left(3\sqrt{6}\right)\left(2\sqrt{2}\right) = 6\sqrt{12} = 6\sqrt{4 \cdot 3} = 12\sqrt{3}$

26. $\left(5\sqrt{8}\right)\left(-3\sqrt{3}\right) = -15\sqrt{24} = -30\sqrt{6}$

27. $3\sqrt{2} + 4\sqrt{2} = (3+4)\sqrt{2} = 7\sqrt{2}$

28. $6\sqrt{5} - 4\sqrt{5} = (6-4)\sqrt{5} = 2\sqrt{5}$

29. $-\sqrt{18} + 2\sqrt{8} = -\sqrt{9 \cdot 2} + 2\sqrt{4 \cdot 2}$

$= -3\sqrt{2} + 4\sqrt{2}$

$= (-3+4)\sqrt{2}$

$= \sqrt{2}$

30. $2\sqrt{12} - 3\sqrt{27} = 2\sqrt{4 \cdot 3} - 3\sqrt{9 \cdot 3}$

$= 4\sqrt{3} - 9\sqrt{3}$

$= (4-9)\sqrt{3}$

$= -5\sqrt{3}$

31. $\left(\sqrt{3}+3\right)\left(\sqrt{3}-1\right) = \left(\sqrt{3}\right)^2 + 3\sqrt{3} - \sqrt{3} - 3$

$= 3 + 2\sqrt{3} - 3$

$= 2\sqrt{3}$

32. $\left(\sqrt{5}-2\right)\left(\sqrt{5}+3\right) = \left(\sqrt{5}\right)^2 - 2\sqrt{5} + 3\sqrt{5} - 6$

$= 5 + \sqrt{5} - 6$

$= \sqrt{5} - 1$

33. $5\sqrt[3]{2}-2\sqrt[3]{54}=5\sqrt[3]{2}-2\cdot 3\sqrt[3]{2}$
$=5\sqrt[3]{2}-6\sqrt[3]{2}$
$=(5-6)\sqrt[3]{2}$
$=-\sqrt[3]{2}$

34. $9\sqrt[3]{24}-\sqrt[3]{81}=9\cdot 2\sqrt[3]{3}-3\sqrt[3]{3}$
$=18\sqrt[3]{3}-3\sqrt[3]{3}$
$=(18-3)\sqrt[3]{3}$
$=15\sqrt[3]{3}$

35. $\left(\sqrt{x}-1\right)^2=\left(\sqrt{x}\right)^2-2\sqrt{x}+1$
$=x-2\sqrt{x}+1$

36. $\left(\sqrt{x}+\sqrt{5}\right)^2=\left(\sqrt{x}\right)^2+2\left(\sqrt{x}\right)\left(\sqrt{5}\right)+\left(\sqrt{5}\right)^2$
$=x+2\sqrt{5x}+5$

37. $\sqrt[3]{16x^4}-\sqrt[3]{2x}=\sqrt[3]{8x^3\cdot 2x}-\sqrt[3]{2x}$
$=2x\sqrt[3]{2x}-\sqrt[3]{2x}$
$=(2x-1)\sqrt[3]{2x}$

38. $\sqrt[4]{32x}+\sqrt[4]{2x^5}=\sqrt[4]{16\cdot 2x}+\sqrt[4]{x^4\cdot 2x}$
$=2\sqrt[4]{2x}+x\sqrt[4]{2x}$
$=(2+x)\sqrt[4]{2x}$ or $(x+2)\sqrt[4]{2x}$

39. $\sqrt{8x^3}-3\sqrt{50x}=\sqrt{4x^2\cdot 2x}-3\sqrt{25\cdot 2x}$
$=2x\sqrt{2x}-15\sqrt{2x}$
$=(2x-15)\sqrt{2x}$

40. $3x\sqrt{9y}+4\sqrt{25y}=9x\sqrt{y}+20\sqrt{y}$
$=(9x+20)\sqrt{y}$

41. $\sqrt[3]{16x^4y}-3x\sqrt[3]{2xy}+5\sqrt[3]{-2xy^4}$
$=\sqrt[3]{8x^3\cdot 2xy}-3x\sqrt[3]{2xy}+5\sqrt[3]{-y^3\cdot 2xy}$
$=2x\sqrt[3]{2xy}-3x\sqrt[3]{2xy}-5y\sqrt[3]{2xy}$
$=(2x-3x-5y)\sqrt[3]{2xy}$
$=(-x-5y)\sqrt[3]{2xy}$ or $-(x+5y)\sqrt[3]{2xy}$

42. $8xy-\sqrt{25x^2y^2}+\sqrt[3]{8x^3y^3}=8xy-5xy+2xy$
$=(8-5+2)xy$
$=5xy$

43. $\dfrac{1}{\sqrt{2}}=\dfrac{1}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$

44. $\dfrac{2}{\sqrt{3}}=\dfrac{2}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{2\sqrt{3}}{3}$

45. $\dfrac{-\sqrt{3}}{\sqrt{5}}=\dfrac{-\sqrt{3}}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{-\sqrt{15}}{5}$

46. $\dfrac{-\sqrt{3}}{\sqrt{8}}=\dfrac{-\sqrt{3}}{2\sqrt{2}}=\dfrac{-\sqrt{3}}{2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{-\sqrt{6}}{2\cdot 2}=\dfrac{-\sqrt{6}}{4}$

47. $\dfrac{\sqrt{3}}{5-\sqrt{2}}=\dfrac{\sqrt{3}}{5-\sqrt{2}}\cdot\dfrac{5+\sqrt{2}}{5+\sqrt{2}}$
$=\dfrac{\sqrt{3}\left(5+\sqrt{2}\right)}{25-2}$
$=\dfrac{\sqrt{3}\left(5+\sqrt{2}\right)}{23}$ or $\dfrac{5\sqrt{3}+\sqrt{6}}{23}$

48. $\dfrac{\sqrt{2}}{\sqrt{7}+2}=\dfrac{\sqrt{2}}{\sqrt{7}+2}\cdot\dfrac{\sqrt{7}-2}{\sqrt{7}-2}$
$=\dfrac{\sqrt{2}\left(\sqrt{7}-2\right)}{7-4}$
$=\dfrac{\sqrt{2}\left(\sqrt{7}-2\right)}{3}$ or $\dfrac{\sqrt{14}-2\sqrt{2}}{3}$

49. $\dfrac{2-\sqrt{5}}{2+3\sqrt{5}}=\dfrac{2-\sqrt{5}}{2+3\sqrt{5}}\cdot\dfrac{2-3\sqrt{5}}{2-3\sqrt{5}}$
$=\dfrac{4-2\sqrt{5}-6\sqrt{5}+15}{4-45}$
$=\dfrac{19-8\sqrt{5}}{-41}=\dfrac{8\sqrt{5}-19}{41}$

50. $\dfrac{\sqrt{3}-1}{2\sqrt{3}+3}=\dfrac{\sqrt{3}-1}{2\sqrt{3}+3}\cdot\dfrac{2\sqrt{3}-3}{2\sqrt{3}-3}$
$=\dfrac{6-2\sqrt{3}-3\sqrt{3}+3}{12-9}=\dfrac{9-5\sqrt{3}}{3}$

51. $\dfrac{5}{\sqrt[3]{2}}=\dfrac{5}{\sqrt[3]{2}}\cdot\dfrac{\sqrt[3]{4}}{\sqrt[3]{4}}=\dfrac{5\sqrt[3]{4}}{2}$

52. $\dfrac{-2}{\sqrt[3]{9}} = \dfrac{-2}{\sqrt[3]{9}} \cdot \dfrac{\sqrt[3]{3}}{\sqrt[3]{3}} = \dfrac{-2\sqrt[3]{3}}{3}$

53. $\dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} \cdot \dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}-\sqrt{x}}$

$$= \frac{(x+h)-2\sqrt{x(x+h)}+x}{(x+h)-x}$$

$$= \frac{x+h-2\sqrt{x^2+xh}+x}{x+h-x}$$

$$= \frac{2x+h-2\sqrt{x^2+xh}}{h}$$

54. $\dfrac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}-\sqrt{x-h}}$

$$= \frac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}-\sqrt{x-h}} \cdot \frac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}+\sqrt{x-h}}$$

$$= \frac{(x+h)+2\sqrt{(x-h)(x+h)}+(x-h)}{(x+h)-(x-h)}$$

$$= \frac{x+h+2\sqrt{x^2-h^2}+x-h}{x+h-x+h}$$

$$= \frac{2x+2\sqrt{x^2-h^2}}{2h}$$

$$= \frac{x+\sqrt{x^2-h^2}}{h}$$

55. $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

56. $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

57. $(-27)^{1/3} = \sqrt[3]{-27} = -3$

58. $16^{3/4} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

59. $16^{3/2} = \left(\sqrt{16}\right)^3 = 4^3 = 64$

60. $25^{3/2} = \left(\sqrt{25}\right)^3 = 5^3 = 125$

61. $9^{-3/2} = \dfrac{1}{9^{3/2}} = \dfrac{1}{\left(\sqrt{9}\right)^3} = \dfrac{1}{3^3} = \dfrac{1}{27}$

62. $16^{-3/2} = \dfrac{1}{16^{3/2}} = \dfrac{1}{\left(\sqrt{16}\right)^3} = \dfrac{1}{4^3} = \dfrac{1}{64}$

63. $\left(\dfrac{9}{8}\right)^{3/2} = \left(\sqrt{\dfrac{9}{8}}\right)^3 = \left(\dfrac{3}{2\sqrt{2}}\right)^3 = \dfrac{3^3}{2^3\left(\sqrt{2}\right)^3}$

$$= \frac{27}{8 \cdot 2\sqrt{2}} = \frac{27}{16\sqrt{2}} = \frac{27}{16\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{27\sqrt{2}}{32}$$

64. $\left(\dfrac{27}{8}\right)^{2/3} = \left(\sqrt[3]{\dfrac{27}{8}}\right)^2 = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

65. $\left(\dfrac{8}{9}\right)^{-3/2} = \left(\dfrac{9}{8}\right)^{3/2} = \left(\sqrt{\dfrac{9}{8}}\right)^3 = \left(\dfrac{3}{2\sqrt{2}}\right)^3$

$$= \frac{3^3}{2^3\left(\sqrt{2}\right)^3} = \frac{27}{8 \cdot 2\sqrt{2}} = \frac{27}{16\sqrt{2}}$$

$$= \frac{27}{16\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{27\sqrt{2}}{32}$$

66. $\left(\dfrac{8}{27}\right)^{-2/3} = \left(\dfrac{27}{8}\right)^{2/3} = \left(\sqrt[3]{\dfrac{27}{8}}\right)^2 = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

67. $x^{3/4}x^{1/3}x^{-1/2} = x^{3/4+1/3-1/2} = x^{7/12}$

68. $x^{2/3}x^{1/2}x^{-1/4} = x^{2/3+1/2-1/4} = x^{11/12}$

69. $\left(x^3y^6\right)^{1/3} = \left(x^3\right)^{1/3}\left(y^6\right)^{1/3} = xy^2$

70. $\left(x^4y^8\right)^{3/4} = \left(x^4\right)^{3/4}\left(y^8\right)^{3/4} = x^3y^6$

71. $\dfrac{\left(x^2y\right)^{1/3}\left(xy^2\right)^{2/3}}{x^{2/3}y^{2/3}} = \dfrac{\left(x^2\right)^{1/3}(y)^{1/3}(x)^{2/3}\left(y^2\right)^{2/3}}{x^{2/3}y^{2/3}}$

$$= \frac{x^{2/3}y^{1/3}x^{2/3}y^{4/3}}{x^{2/3}y^{2/3}}$$

$$= x^{2/3+2/3-2/3}y^{1/3+4/3-2/3}$$

$$= x^{2/3}y^1 = x^{2/3}y$$

72. $$\frac{(xy)^{1/4}\left(x^2y^2\right)^{1/2}}{\left(x^2y\right)^{3/4}} = \frac{x^{1/4}y^{1/4}\left(x^2\right)^{1/2}\left(y^2\right)^{1/2}}{\left(x^2\right)^{3/4}y^{3/4}}$$
$$= \frac{x^{1/4}y^{1/4}xy}{x^{3/2}y^{3/4}}$$
$$= x^{1/4+1-3/2}y^{1/4+1-3/4}$$
$$= x^{-1/4}y^{1/2} = \frac{y^{1/2}}{x^{1/4}}$$

73. $$\frac{\left(16x^2y^{-1/3}\right)^{3/4}}{\left(xy^2\right)^{1/4}} = \frac{16^{3/4}\left(x^2\right)^{3/4}\left(y^{-1/3}\right)^{3/4}}{x^{1/4}\left(y^2\right)^{1/4}}$$
$$= \frac{\left(\sqrt[4]{16}\right)^3 x^{3/2}y^{-1/4}}{x^{1/4}y^{1/2}}$$
$$= 2^3x^{3/2-1/4}y^{-1/4-1/2}$$
$$= 8x^{5/4}y^{-3/4}$$
$$= \frac{8x^{5/4}}{y^{3/4}}$$

74. $$\frac{\left(4x^{-1}y^{1/3}\right)^{3/2}}{\left(xy\right)^{3/2}} = \frac{4^{3/2}\left(x^{-1}\right)^{3/2}\left(y^{1/3}\right)^{3/2}}{x^{3/2}y^{3/2}}$$
$$= \frac{\left(\sqrt{4}\right)^3 x^{-3/2}y^{1/2}}{x^{3/2}y^{3/2}}$$
$$= 2^3x^{-3/2-3/2}y^{1/2-3/2}$$
$$= 8x^{-3}y^{-1}$$
$$= \frac{8}{x^3y}$$

75. $$\frac{x}{(1+x)^{1/2}} + 2(1+x)^{1/2} = \frac{x+2(1+x)^{1/2}(1+x)^{1/2}}{(1+x)^{1/2}}$$
$$= \frac{x+2(1+x)}{(1+x)^{1/2}}$$
$$= \frac{x+2+2x}{(1+x)^{1/2}}$$
$$= \frac{3x+2}{(1+x)^{1/2}}$$

76. $$\frac{1+x}{2x^{1/2}} + x^{1/2} = \frac{1+x+x^{1/2}\cdot 2x^{1/2}}{2x^{1/2}}$$
$$= \frac{1+x+2x}{2x^{1/2}} = \frac{3x+1}{2x^{1/2}}$$

77. $$2x\left(x^2+1\right)^{1/2} + x^2\cdot\frac{1}{2}\left(x^2+1\right)^{-1/2}\cdot 2x$$
$$= 2x\left(x^2+1\right)^{1/2} + \frac{x^3}{\left(x^2+1\right)^{1/2}}$$
$$= \frac{2x\left(x^2+1\right)^{1/2}\cdot\left(x^2+1\right)^{1/2} + x^3}{\left(x^2+1\right)^{1/2}}$$
$$= \frac{2x\left(x^2+1\right)^{1/2+1/2} + x^3}{\left(x^2+1\right)^{1/2}} = \frac{2x\left(x^2+1\right)^1 + x^3}{\left(x^2+1\right)^{1/2}}$$
$$= \frac{2x^3+2x+x^3}{\left(x^2+1\right)^{1/2}} = \frac{3x^3+2x}{\left(x^2+1\right)^{1/2}}$$
$$= \frac{x\left(3x^2+2\right)}{\left(x^2+1\right)^{1/2}}$$

78. $$(x+1)^{1/3} + x\cdot\frac{1}{3}(x+1)^{-2/3}, x \neq -1$$
$$= (x+1)^{1/3} + \frac{x}{3(x+1)^{2/3}}$$
$$= \frac{3(x+1)^{2/3}(x+1)^{1/3} + x}{3(x+1)^{2/3}}$$
$$= \frac{3(x+1)^{2/3+1/3} + x}{3(x+1)^{2/3}} = \frac{3(x+1)^1 + x}{3(x+1)^{2/3}}$$
$$= \frac{3x+3+x}{3(x+1)^{2/3}} = \frac{4x+3}{3(x+1)^{2/3}}$$

79. $\sqrt{4x+3}\cdot\frac{1}{2\sqrt{x-5}}+\sqrt{x-5}\cdot\frac{1}{5\sqrt{4x+3}}, x>5$

$$=\frac{\sqrt{4x+3}}{2\sqrt{x-5}}+\frac{\sqrt{x-5}}{5\sqrt{4x+3}}$$
$$=\frac{\sqrt{4x+3}\cdot 5\cdot\sqrt{4x+3}+\sqrt{x-5}\cdot 2\cdot\sqrt{x-5}}{10\sqrt{x-5}\sqrt{4x+3}}$$
$$=\frac{5(4x+3)+2(x-5)}{10\sqrt{(x-5)(4x+3)}}$$
$$=\frac{20x+15+2x-10}{10\sqrt{(x-5)(4x+3)}}$$
$$=\frac{22x+5}{10\sqrt{(x-5)(4x+3)}}$$

80. $\frac{\sqrt[3]{8x+1}}{3\sqrt[3]{(x-2)^2}}+\frac{\sqrt[3]{x-2}}{24\sqrt[3]{(8x+1)^2}}, x\neq 2, x\neq -\frac{1}{8}$

$$=\frac{8\sqrt[3]{8x+1}\cdot\sqrt[3]{(8x+1)^2}+\sqrt[3]{x-2}\cdot\sqrt[3]{(x-2)^2}}{24\sqrt[3]{(x-2)^2}\cdot\sqrt[3]{(8x+1)^2}}$$
$$=\frac{8\sqrt[3]{(8x+1)^3}+\sqrt[3]{(x-2)^3}}{24\sqrt[3]{(x-2)^2}\cdot\sqrt[3]{(8x+1)^2}}$$
$$=\frac{8(8x+1)+x-2}{24\sqrt[3]{(x-2)^2(8x+1)^2}}$$
$$=\frac{64x+8+x-2}{24\sqrt[3]{(x-2)^2(8x+1)^2}}$$
$$=\frac{65x+6}{24\sqrt[3]{(x-2)^2(8x+1)^2}}$$

81. $\frac{\left(\sqrt{1+x}-x\cdot\frac{1}{2\sqrt{1+x}}\right)}{1+x}=\frac{\left(\sqrt{1+x}-\frac{x}{2\sqrt{1+x}}\right)}{1+x}$

$$=\frac{\left(\frac{2\sqrt{1+x}\sqrt{1+x}-x}{2\sqrt{1+x}}\right)}{1+x}$$
$$=\frac{2(1+x)-x}{2(1+x)^{1/2}}\cdot\frac{1}{1+x}$$
$$=\frac{2+x}{2(1+x)^{3/2}}$$

82. $\frac{\left(\sqrt{x^2+1}-x\cdot\frac{2x}{2\sqrt{x^2+1}}\right)}{x^2+1}$

$$=\frac{\left(\sqrt{x^2+1}-\frac{x^2}{\sqrt{x^2+1}}\right)}{x^2+1}$$
$$=\frac{\left(\sqrt{x^2+1}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}-\frac{x^2}{\sqrt{x^2+1}}\right)}{x^2+1}$$
$$=\frac{\left(\frac{x^2+1-x^2}{\sqrt{x^2+1}}\right)}{x^2+1}=\frac{1}{\sqrt{x^2+1}}\cdot\frac{1}{x^2+1}$$
$$=\frac{1}{(x^2+1)^{3/2}}$$

83. $\frac{(x+4)^{1/2}-2x(x+4)^{-1/2}}{x+4}$

$$=\frac{\left((x+4)^{1/2}-\frac{2x}{(x+4)^{1/2}}\right)}{x+4}$$
$$=\frac{\left((x+4)^{1/2}\cdot\frac{(x+4)^{1/2}}{(x+4)^{1/2}}-\frac{2x}{(x+4)^{1/2}}\right)}{x+4}$$
$$=\frac{\left(\frac{x+4-2x}{(x+4)^{1/2}}\right)}{x+4}$$
$$=\frac{-x+4}{(x+4)^{1/2}}\cdot\frac{1}{x+4}$$
$$=\frac{-x+4}{(x+4)^{3/2}}$$
$$=\frac{4-x}{(x+4)^{3/2}}$$

84. $\dfrac{\left(9-x^2\right)^{1/2}+x^2\left(9-x^2\right)^{-1/2}}{9-x^2}, -3<x<3$

$$\frac{\left(\left(9-x^2\right)^{1/2}+\dfrac{x^2}{\left(9-x^2\right)^{1/2}}\right)}{9-x^2}$$

$$=\frac{\left(\dfrac{\left(9-x^2\right)^{1/2}\cdot\left(9-x^2\right)^{1/2}+x^2}{\left(9-x^2\right)^{1/2}}\right)}{9-x^2}$$

$$=\frac{\left(9-x^2\right)^{1/2}\cdot\left(9-x^2\right)^{1/2}+x^2}{\left(9-x^2\right)^{1/2}}\cdot\frac{1}{9-x^2}$$

$$=\frac{9-x^2+x^2}{\left(9-x^2\right)^{1/2}}\cdot\frac{1}{9-x^2}$$

$$=\frac{9}{\left(9-x^2\right)^{3/2}}$$

85. $\dfrac{\dfrac{x^2}{\left(x^2-1\right)^{1/2}}-\left(x^2-1\right)^{1/2}}{x^2}, x<-1 \text{ or } x>1$

$$=\frac{\left(\dfrac{x^2-\left(x^2-1\right)^{1/2}\cdot\left(x^2-1\right)^{1/2}}{\left(x^2-1\right)^{1/2}}\right)}{x^2}$$

$$=\frac{x^2-\left(x^2-1\right)^{1/2}\cdot\left(x^2-1\right)^{1/2}}{\left(x^2-1\right)^{1/2}}\cdot\frac{1}{x^2}$$

$$=\frac{x^2-\left(x^2-1\right)}{\left(x^2-1\right)^{1/2}}\cdot\frac{1}{x^2}$$

$$=\frac{x^2-x^2+1}{\left(x^2-1\right)^{1/2}}\cdot\frac{1}{x^2}$$

$$=\frac{1}{x^2\left(x^2-1\right)^{1/2}}$$

86. $\dfrac{\left(x^2+4\right)^{1/2}-x^2\left(x^2+4\right)^{-1/2}}{x^2+4}$

$$=\frac{\left(\left(x^2+4\right)^{1/2}-\dfrac{x^2}{\left(x^2+4\right)^{1/2}}\right)}{x^2+4}$$

$$=\frac{\left(\dfrac{\left(x^2+4\right)^{1/2}\cdot\left(x^2+4\right)^{1/2}-x^2}{\left(x^2+4\right)^{1/2}}\right)}{x^2+4}$$

$$=\frac{\left(x^2+4\right)^{1/2}\cdot\left(x^2+4\right)^{1/2}-x^2}{\left(x^2+4\right)^{1/2}}\cdot\frac{1}{x^2+4}$$

$$=\frac{x^2+4-x^2}{\left(x^2+4\right)^{1/2}}\cdot\frac{1}{x^2+4}=\frac{4}{\left(x^2+4\right)^{3/2}}$$

87. $\dfrac{\dfrac{1+x^2}{2\sqrt{x}}-2x\sqrt{x}}{\left(1+x^2\right)^2}, x>0$

$$=\frac{\left(\dfrac{1+x^2-\left(2\sqrt{x}\right)\left(2x\sqrt{x}\right)}{2\sqrt{x}}\right)}{\left(1+x^2\right)^2}$$

$$=\frac{1+x^2-\left(2\sqrt{x}\right)\left(2x\sqrt{x}\right)}{2\sqrt{x}}\cdot\frac{1}{\left(1+x^2\right)^2}$$

$$=\frac{1+x^2-4x^2}{2\sqrt{x}}\cdot\frac{1}{\left(1+x^2\right)^2}=\frac{1-3x^2}{2\sqrt{x}\left(1+x^2\right)^2}$$

88. $\dfrac{2x\left(1-x^2\right)^{1/3}+\dfrac{2}{3}x^3\left(1-x^2\right)^{-2/3}}{\left(1-x^2\right)^{2/3}}, x\neq -1, x\neq 1$

$$=\frac{\left(2x\left(1-x^2\right)^{1/3}+\dfrac{2x^3}{3\left(1-x^2\right)^{2/3}}\right)}{\left(1-x^2\right)^{2/3}}$$

$$=\frac{\left(\dfrac{2x\left(1-x^2\right)^{1/3}3\left(1-x^2\right)^{2/3}+2x^3}{3\left(1-x^2\right)^{2/3}}\right)}{\left(1-x^2\right)^{2/3}}$$

$$= \frac{6x\left(1-x^2\right)^{1/3+2/3}+2x^3}{3\left(1-x^2\right)^{2/3}} \cdot \frac{1}{\left(1-x^2\right)^{2/3}}$$

$$= \frac{6x\left(1-x^2\right)+2x^3}{3\left(1-x^2\right)^{2/3+2/3}} = \frac{6x-6x^3+2x^3}{3\left(1-x^2\right)^{4/3}}$$

$$= \frac{6x-4x^3}{3\left(1-x^2\right)^{4/3}} = \frac{2x\left(3-2x^2\right)}{3\left(1-x^2\right)^{4/3}}$$

89. $(x+1)^{3/2} + x \cdot \frac{3}{2}(x+1)^{1/2}$

$$= (x+1)^{1/2}\left(x+1+\frac{3}{2}x\right)$$

$$= (x+1)^{1/2}\left(\frac{5}{2}x+1\right)$$

$$= \frac{1}{2}(x+1)^{1/2}(5x+2)$$

90. $(x^2+4)^{4/3} + x \cdot \frac{4}{3}(x^2+4)^{1/3} \cdot 2x$

$$= (x^2+4)^{1/3}\left(x^2+4+\frac{8}{3}x^2\right)$$

$$= (x^2+4)^{1/3}\left(\frac{11}{3}x^2+4\right)$$

$$= \frac{1}{3}\left(x^2+4\right)^{1/3}\left(11x^2+12\right)$$

91. $6x^{1/2}\left(x^2+x\right) - 8x^{3/2} - 8x^{1/2}$

$$= 2x^{1/2}\left(3(x^2+x)-4x-4\right)$$

$$= 2x^{1/2}\left(3x^2-x-4\right)$$

$$= 2x^{1/2}(3x-4)(x+1)$$

92. $6x^{1/2}(2x+3) + x^{3/2} \cdot 8$

$$= 2x^{1/2}\left(3(2x+3)+4x\right)$$

$$= 2x^{1/2}(10x+9)$$

93. $3\left(x^2+4\right)^{4/3} + x \cdot 4\left(x^2+4\right)^{1/3} \cdot 2x$

$$= \left(x^2+4\right)^{1/3}\left[3\left(x^2+4\right)+8x^2\right]$$

$$= \left(x^2+4\right)^{1/3}\left[3x^2+12+8x^2\right]$$

$$= \left(x^2+4\right)^{1/3}\left(11x^2+12\right)$$

94. $2x(3x+4)^{4/3} + x^2 \cdot 4(3x+4)^{1/3}$

$$= 2x(3x+4)^{1/3}\left[(3x+4)+2x\right]$$

$$= 2x(3x+4)^{1/3}(5x+4)$$

95. $4(3x+5)^{1/3}(2x+3)^{3/2} + 3(3x+5)^{4/3}(2x+3)^{1/2}$

$$= (3x+5)^{1/3}(2x+3)^{1/2}\left[4(2x+3)+3(3x+5)\right]$$

$$= (3x+5)^{1/3}(2x+3)^{1/2}(8x+12+9x+15)$$

$$= (3x+5)^{1/3}(2x+3)^{1/2}(17x+27)$$

where $x \geq -\frac{3}{2}$.

96. $6(6x+1)^{1/3}(4x-3)^{3/2} + 6(6x+1)^{4/3}(4x-3)^{1/2}$

$$= 6(6x+1)^{1/3}(4x-3)^{1/2}\left[(4x-3)+(6x+1)\right]$$

$$= 6(6x+1)^{1/3}(4x-3)^{1/2}(10x-2)$$

$$= 6(6x+1)^{1/3}(4x-3)^{1/2}(2)(5x-1)$$

$$= 12(6x+1)^{1/3}(4x-3)^{1/2}(5x-1)$$

where $x \geq \frac{3}{4}$.

97. $3x^{-1/2} + \frac{3}{2}x^{1/2}, x > 0$

$$= \frac{3}{x^{1/2}} + \frac{3}{2}x^{1/2}$$

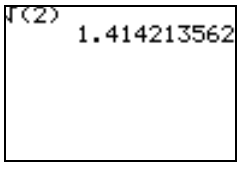
$$= \frac{3 \cdot 2 + 3x^{1/2} \cdot x^{1/2}}{2x^{1/2}} = \frac{6+3x}{2x^{1/2}} = \frac{3(x+2)}{2x^{1/2}}$$

98. $8x^{1/3} - 4x^{-2/3}, x \neq 0$

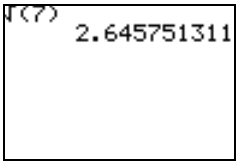
$$= 8x^{1/3} - \frac{4}{x^{2/3}}$$

$$= \frac{8x^{1/3} \cdot x^{2/3} - 4}{x^{2/3}} = \frac{8x-4}{x^{2/3}} = \frac{4(2x-1)}{x^{2/3}}$$

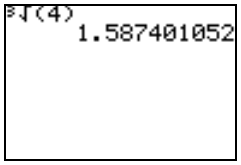
99. $\sqrt{2} \approx 1.41$



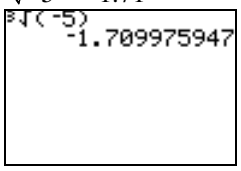
100. $\sqrt{7} \approx 2.65$



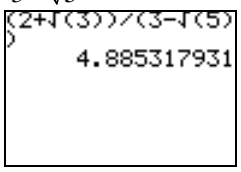
101. $\sqrt[3]{4} \approx 1.59$



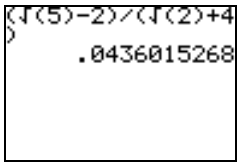
102. $\sqrt[3]{-5} \approx -1.71$



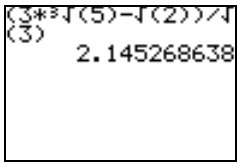
103. $\dfrac{2+\sqrt{3}}{3-\sqrt{5}} \approx 4.89$



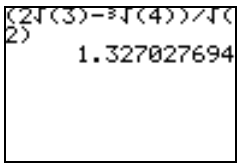
104. $\dfrac{\sqrt{5}-2}{\sqrt{2}+4} \approx 0.04$



105. $\dfrac{3\sqrt[3]{5}-\sqrt{2}}{\sqrt{3}} \approx 2.15$



106. $\dfrac{2\sqrt{3}-\sqrt[3]{4}}{\sqrt{2}} \approx 1.33$



107. **a.** $V = 40(12)^2\sqrt{\dfrac{96}{12}-0.608}$

$\approx 15{,}660.4$ gallons

b. $V = 40(1)^2\sqrt{\dfrac{96}{1}-0.608} \approx 390.7$ gallons

108. **a.** $v = \sqrt{64\cdot 4+0^2} = \sqrt{256}$

$= 16$ feet per second

b. $v = \sqrt{64\cdot 16+0^2} = \sqrt{1024}$

$= 32$ feet per second

c. $v = \sqrt{64\cdot 2+4^2} = \sqrt{144}$

$= 12$ feet per second

109. $T = 2\pi\sqrt{\dfrac{64}{32}} = 2\pi\sqrt{2} \approx 8.89$ seconds

110. $T = 2\pi\sqrt{\dfrac{16}{32}} = 2\pi\sqrt{\dfrac{1}{2}} = \dfrac{2\pi}{\sqrt{2}}$

$= \pi\sqrt{2} \approx 4.44$ seconds

111. 8 inches = 8/12 = 2/3 feet

$$T = 2\pi\sqrt{\frac{\left(\frac{2}{3}\right)}{32}} = 2\pi\sqrt{\frac{1}{48}} = 2\pi\left(\frac{1}{4\sqrt{3}}\right)$$

$$= \frac{\pi}{2\sqrt{3}} = \frac{\pi\sqrt{3}}{6} \approx 0.91 \text{ seconds}$$

112. 4 inches = 4/12 = 1/3 feet

$$T = 2\pi\sqrt{\frac{\left(\frac{1}{3}\right)}{32}} = 2\pi\sqrt{\frac{1}{96}} = 2\pi\left(\frac{1}{4\sqrt{6}}\right)$$

$$= \frac{\pi}{2\sqrt{6}} = \frac{\pi\sqrt{6}}{12} \approx 0.64 \text{ seconds}$$

113. Answers may vary. One possibility follows: If $a = -5$, then $\sqrt{a^2} = \sqrt{(-5)^2} = \sqrt{25} = 5 \neq a$.

Since we use the principal square root, which is always non-negative,

$$\sqrt{a^2} = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

which is the definition of $|a|$, so $\sqrt{a^2} = |a|$.